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NON-LINEAR VISCOELASTIC BEHAVIOR OF SOME POLYMERS AND THEIR COMPOSITES

FREDERIC SHRIVER HERING

NON-LINEAR VISCOELASTIC BEHAVIOR OF SOME POLYMERS AND THEIR COMPOSITES

by

Frederic Shriver Hering
Lieutenant, United States Navy
B.Met.E., Rensselaer Polytechnic Institute, 1961

[redacted]

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requirements for the degree of

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ABSTRACT

Constant load creep data were obtained for four plastics: polyethyline, polypropylene, nylon, and polycarbonate. Viscoelastic responses of these materials proved to be non-linear in both time and stress. An empirical creep equation, based on a time dependent retardation rate constant, was developed to characterize the creep behavior and good correlation was obtained for various stress levels. By assuming the developed relationship to be an equation of state, creep behavior of simple composites of plastics was predicted with good results when viscoelastic responses of the components were similar. The importance of stress-time history is demonstrated by using the creep equation to estimate mechanical behavior in other modes of testing. The concept of a retardation spectrum and its application to the materials studied is considered.

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NOTATION

τ - shear stress

γ - shear strain

η - viscosity coefficient

$\dot{\gamma}$ - shear strain rate

G - shear modulus

E - tensile modulus

t - time

ϵ - engineering strain

$\dot{\epsilon}$ - strain rate

σ - engineering stress

$\dot{\sigma}$ - stress rate

σ_o - initial stress

σ_i - stress at time "t_i"

$J(t)$ - creep compliance

θ - current time

e - natural logarithm base

T_g - glass transition temperature

$\mathcal{F}$ - free energy of deformation

ϵ_o - zero time strain (instantaneous elastic strain)

ϵ_d - delayed elastic strain

$E_r(t)$ - relaxation modulus

ϵ_f - non-recoverable flow strain

δ - deformation

$\dot{\delta}$ - deformation rate

PE - polyethylene

PP - polypropylene

PC - polycarbonate

NY - nylon

τ - retardation time constant

b - rate constant for delayed strain response

$g(b)$ - rate constant distribution function

$L(\tau)$ - retardation time constant distribution function

ε - delayed strain fraction

A_1 - creep parameter representing zero-time strain

A_2 - creep parameter representing steady-state creep rate

A_3 - creep parameter representing magnitude of delayed elastic strain component

C - time dependent rate constant in empirical creep equation

Q - time exponent in empirical creep equation

K, B, P - stress coefficients in generalized creep equation

N, M, R - stress exponents in generalized creep equation

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1. Introduction.

Composite materials with high strength-to-weight ratios show great promise. [3] Composites may be looked upon as intimate mechanical combinations of two or more materials, but differ from multiphase materials such as metallic alloys in that they are not homogeneous on a macroscopic scale. Composite materials are not new. Concrete and filled resins, for example, have been successfully employed for years, and currently a bewildering number of combinations of many thousands of materials is available. Of necessity, however, the work with composites to date has been directed towards development of materials to meet specific technological requirements, and very little research has been carried out in the area of characterization of the behavior of composites. [1, 2]

Physical properties often constitute the starting point for the materials selection process, and so characterizing mechanical behavior of composites is an important first step in a fundamental research study. Of particular interest is viscoelastic behavior, frequently manifested by the component materials of the composite structure. Deviations from true elasticity are negligible for engineering purposes for materials such as metals and glasses, but in polymeric systems, which are extensively employed in composites, behavior is dominated by viscoelastic phenomena. [2] Many conventional composite materials have mechanical properties that exhibit appreciable time dependence through creep and relaxation processes. [4] The stress analysis becomes quite complicated, but it must be performed if a viscoelastic composite is to be utilized in an efficiently designed structure giving satisfactory performance in service environments. [5]

In this investigation the main purpose was to characterize the creep behavior of individual viscoelastic materials by analytic equations, and then obtain suitable correlations so that the behavior of composites of these materials could be predicted. In addition, these analytic expressions were used to estimate performance of materials in modes of testing other than creep, such as stress relaxation and constant strain rate uniaxial tension tests.

For the experiment, four different thermoplastics (polyethylene, polypropylene, nylon, and polycarbonate) were chosen, and composites were formed as layered sandwich structures. A simple composite was selected for this basic study to eliminate as many variables as possible, since stress analysis of polymeric materials is inherently complex, even if linear viscoelasticity is assumed. In all experiments the specimens were subjected to uniaxial tension, and it has been shown [6] for this type of loading that changing the type and amount of component interfacial contact has negligible effect on composite behavior. This investigation was carried out in a laboratory with controlled temperature of 23 ± 1° C. and relative humidity 50 ± 2% in accordance with ASTM Standards. [7] Laboratory set-up and equipment is reported in Appendix G.

2. Linear Viscoelasticity.

For the ideal elastic solid, stress is proportional to strain when a load is applied in accordance with Hooke's Law. It is also independent of the rate of strain. For example, at stresses below the yield strength, the behavior of most metals is given by Hooke's Law:

$$\tau = G\gamma \tag{2.1}$$

where τ = applied shear stress
γ = shear strain
G = modulus of rigidity

For the ideal viscous liquid, stress is proportional to the rate of strain, but independent of the strain itself, as stated by Newton's Viscosity Law. [8] At low shear stresses most liquids and gases obey Newton's Law:

$$\tau = \eta\dot{\gamma} \tag{2.2}$$

where τ = applied shear stress
$\dot{\gamma}$ = shear strain rate
η = Newtonian viscosity coefficient

The laws and equations above represent idealizations. If sufficiently precise measurements are made it is found that no real solid is truly Hookean and that no real liquid is exactly Newtonian in mechanical response. [9] To represent such deviations an equation can be written in the form:

$$\gamma = B\tau t^{n} \tag{2.3}$$

For the ideal elastic solid, n equals zero and the equation becomes:

$$\gamma = B\tau \tag{2.4}$$

where the constant B is the reciprocal of the shear modulus, $1/G$. For a perfectly viscous liquid, the exponent n equals one, and the equation becomes:

$$\gamma = B\tau t$$

or

$$\dot{\gamma} = Bt \tag{2.5}$$

where the constant B is the reciprocal of the Newtonian Viscosity, $1/\eta$. For a real material the exponent n can vary between zero and one, and is a measure of the relative importance of the elastic and viscous contributions to the mechanical behavior. For values of the exponent n between zero and one, equation (2.3) indicates a time behavior in which stress depends on both strain and strain rate. A material such as this which shows characteristics intermediate between those of an ideal solid or ideal liquid is called viscoelastic.

In some situations neither the strain nor the strain rate is directly proportional to stress. An example of this complicated relationship is the plastic deformation of metals. In the absence of this complexity, and when both strain and strain rate are linear with stress as indicated by equation (2.3) a material is said to display linear viscoelastic behavior. [2]

Linear viscoelasticity might be expected with materials whose structure is not changing during the time it is subjected to an applied stress. [11] For such linear viscoelastic behavior, a law known as the Boltzmann Superposition Principle applies. This principle is illustrated in Figure 1, which relates to a creep test where a material is subjected to a constant stress, and the creep strain changes with time. For linear viscoelastic behavior, this strain is directly proportional to stress:

(2.6) $$\epsilon(t) = \sigma\, J(t)$$

where $J(t)$, the creep compliance, is a function representing the time effects of the viscoelastic response. For a stress σ_0 applied at zero time, the strain as a function of time is:

(2.7) $$\epsilon(t) = \sigma_0\, J(t)$$

For some higher stress $\sigma_0 + \sigma_1$ applied at zero time, the strain would be increased:

(2.8) $$\epsilon(t) = (\sigma_0 + \sigma_1) \cdot J(t)$$

Now if σ_0 is applied at time $t = 0$, and stress σ_1 is added at a later time $t = t_1$ the strain becomes:

(2.9) $$\epsilon(t) = \sigma_0 \cdot J(t) + \sigma_1 \cdot J(t - t_1)$$

This is illustrated in Figure 1 where linear superposition of the strains at their elapsed times is indicated. The stress history may be generalized by:

(2.10) $$\epsilon(t) = \sum_{t_i = -\infty}^{t_i = t} \sigma_i \cdot J(t - t_i)$$

For a continuous stress history starting at zero time, the Boltzmann Superposition Principle can be expressed in integral form:

(2.11) $$\epsilon(t) = \int_{\theta = 0}^{\theta = t} J(t - \theta)\, \dot{\sigma}(\theta)\, d\theta$$

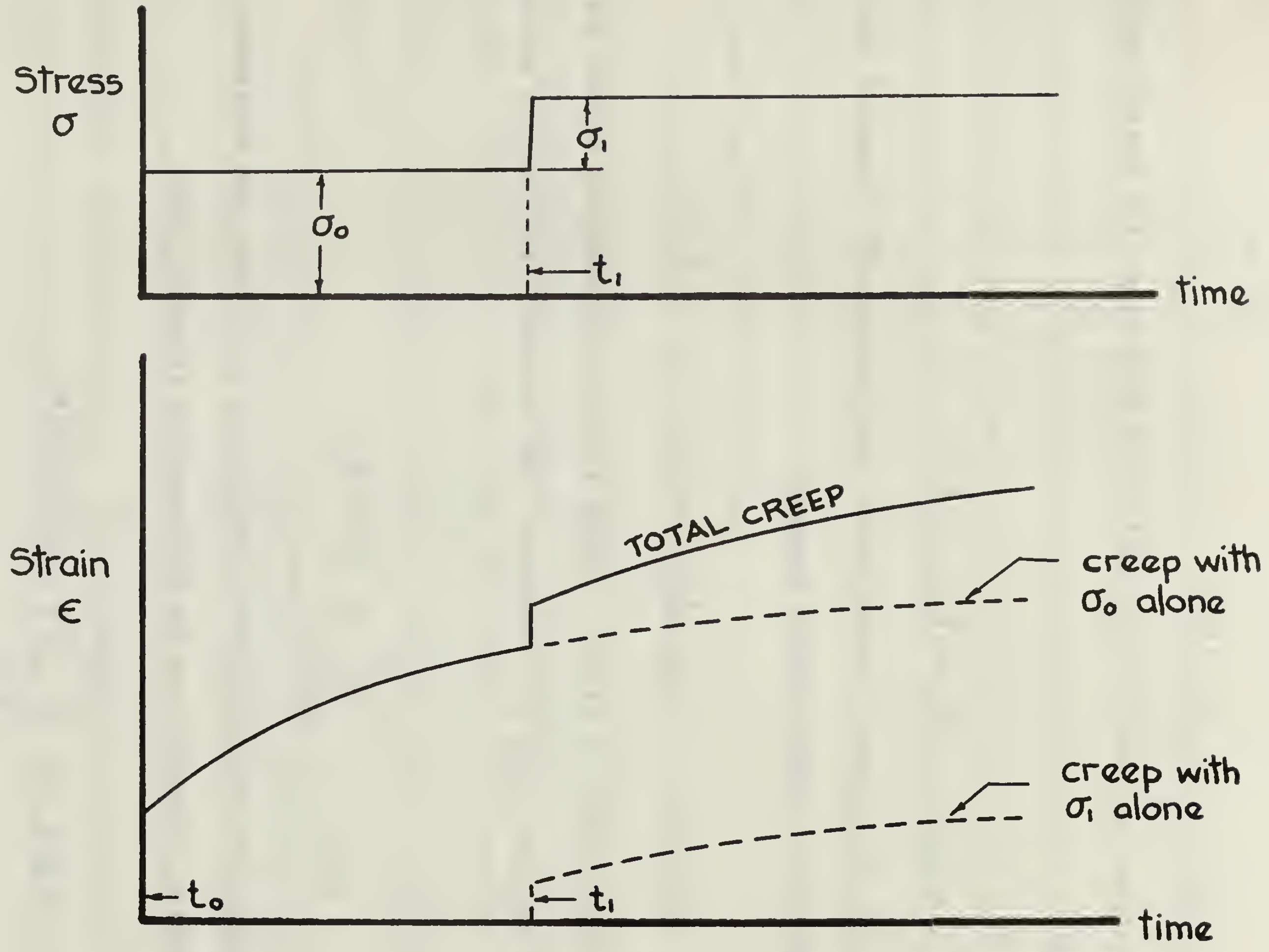

Figure 1

The Boltzmann Superposition Principle Applied to Creep (from Ref. 15) where θ is current time during the loading history, where $\dot{\sigma}(\theta)$ is written for $\frac{d\sigma(\theta)}{d\theta}$, and t is constant in the integration. The creep strain in equation (2.11) is therefore represented by a convolution integral where $\dot{\sigma}(\theta)$ is the "forcing function," and $J(t-\theta)$ is the "response function."

In a stress relaxation test, a tensile strain is rapidly applied and the stress changes with time. For linear viscoelastic behavior,

$$\sigma(t) = \epsilon \cdot E_r(t) \tag{2.12}$$

Here the viscoelastic time behavior is given by $E_r(t)$, called the relaxation modulus. Equations analogous to (2.7) through (2.10) can be written for successively applied strains, so that for a continuous strain history,

$$\sigma(t) = \int_{\theta=0}^{\theta=t} E_r(t-\theta)\,\dot{\epsilon}(\theta)\,d\theta \tag{2.13}$$

It has been shown by Volterra that equations (2.11) and (2.13) are consequences of each other. [11, 12, 13] These equations are also a statement of Boltzmann's Superposition Principle and a definition of linear viscoelastic behavior. A thorough treatment of the mathematical theory and implications of linear viscoelasticity can be found in reference [14].

3. Linear Viscoelastic Models.

Mechanical models are not necessary to describe viscoelastic phenomena, but they have been useful in describing a combination of Hookean and Newtonian characteristics of materials. The elastic mode of stress response may be represented by a spring (Figure 2), presumed to behave in an ideal Hookean manner, i.e., strain proportional to applied stress and independent of strain rate. The flow mechanism is indicated by a dashpot, (Figure 3) assumed to be filled with a Newtonian fluid so that its deformation is proportional to the time and to applied stress, and inversely proportional to the viscosity of the fluid, η . The behavior of a material displaying flow superimposed upon elasticity can be represented by a combination of a dashpot and a spring. The two basic models are the Maxwell Element (Figure 4) and the Voigt Element (Figure 5). The response of these units to an applied stress is illustrated in Figure 6. In order to represent a real material an infinite number of these elements may be required. Generalized Voigt and Maxwell models are shown in Figures 7 and 8. Poincaré [16] indicates that a phenomenon representable by any one model is also representable by an infinite number of other models. If one dashpot in the generalized Maxwell model has infinite viscosity ($\eta = \infty$) and if one dashpot in the generalized Voigt model has zero viscosity ($\eta = 0$) then the models become equivalent in their representation of viscoelastic behavior.

The creep behavior of most materials can be considered as the superposition of three stress responses. (Figure 9) The instantaneous elastic strain can be represented by a Hookean spring, and the non-recoverable viscous flow by a Newtonian dashpot. The Voigt Element is particularly useful in describing a delayed elastic response. The viscous element coupled in parallel with the elastic element represents a damping resistance

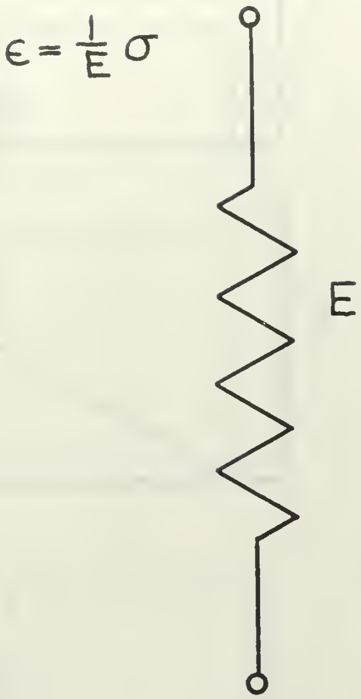

Figure 2

Hookean Solid

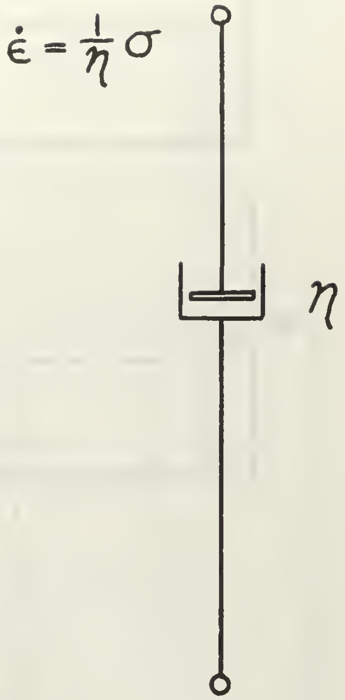

Figure 3

Newtonian Liquid

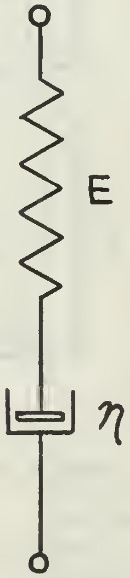

Figure 4

Maxwell Element

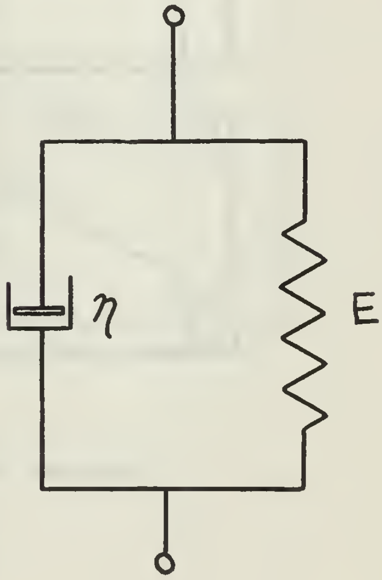

Figure 5

Voigt Element

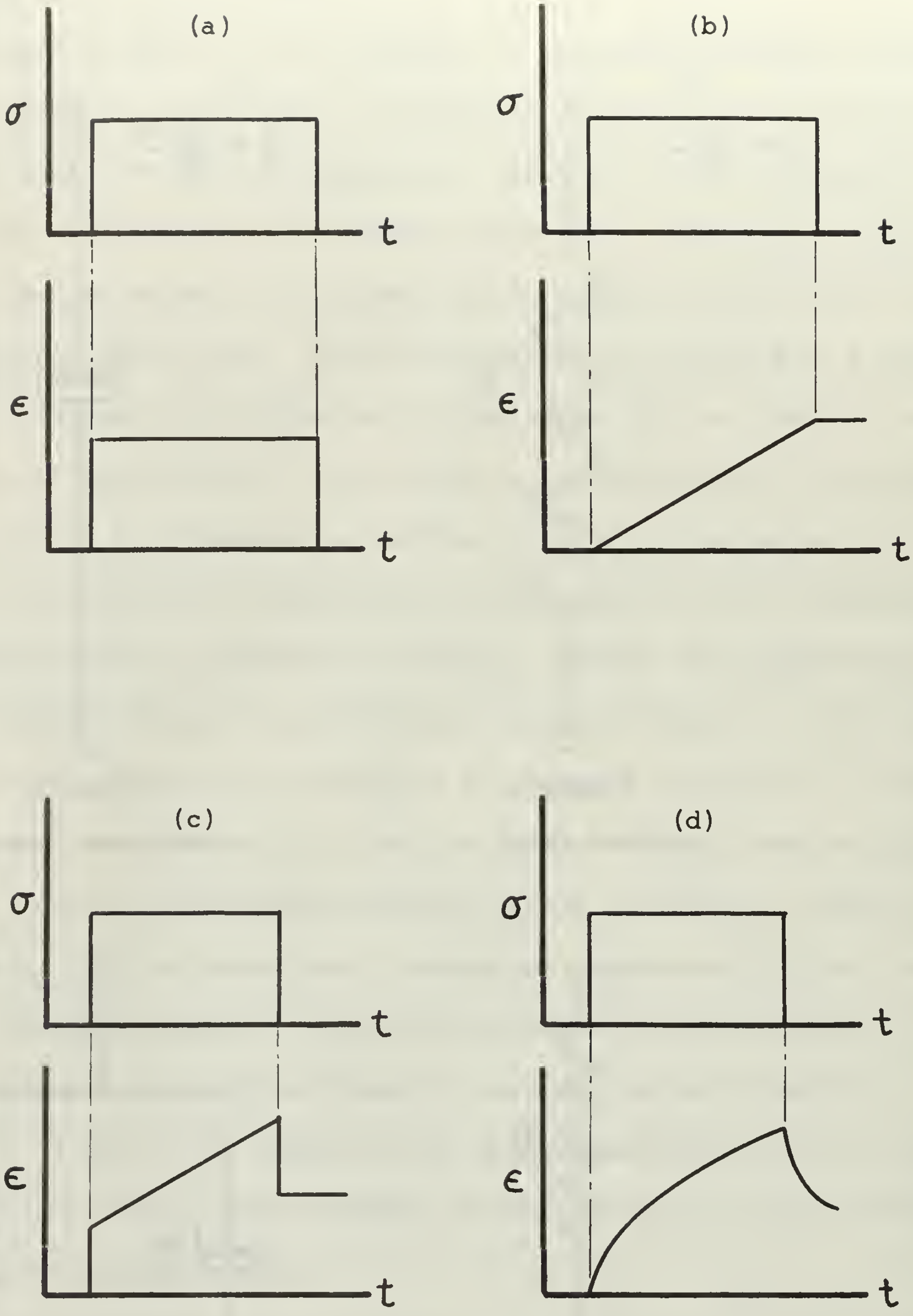

Figure 6

Response of Viscoelastic Models to Stress

(a) Hookean Solid
(b) Newtonian Liquid
(c) Maxwell Element
(d) Voigt Element

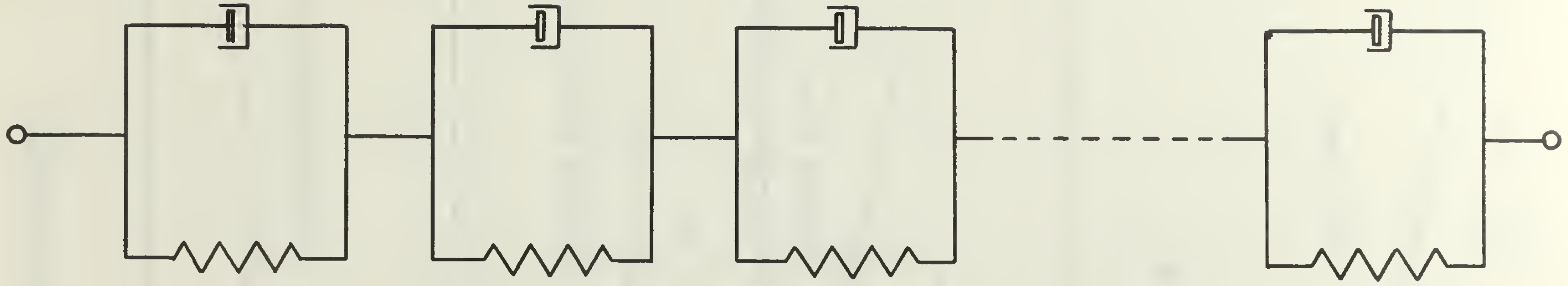

Figure 7

The Generalized Voigt Model

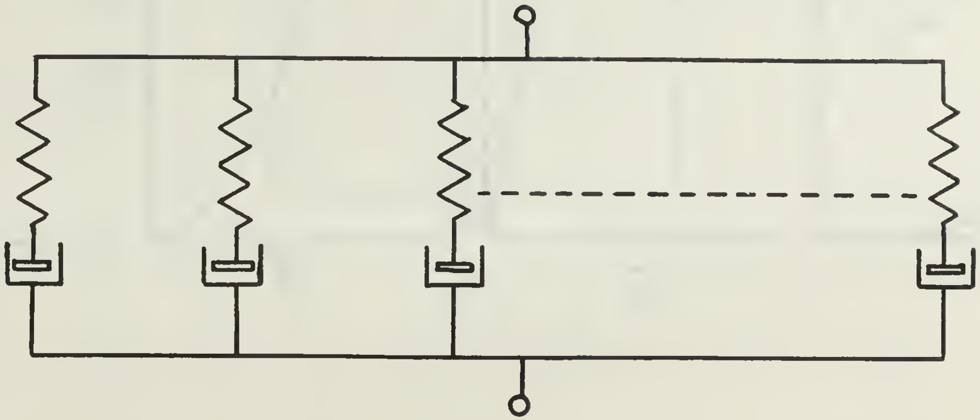

Figure 8

The Generalized Maxwell Model

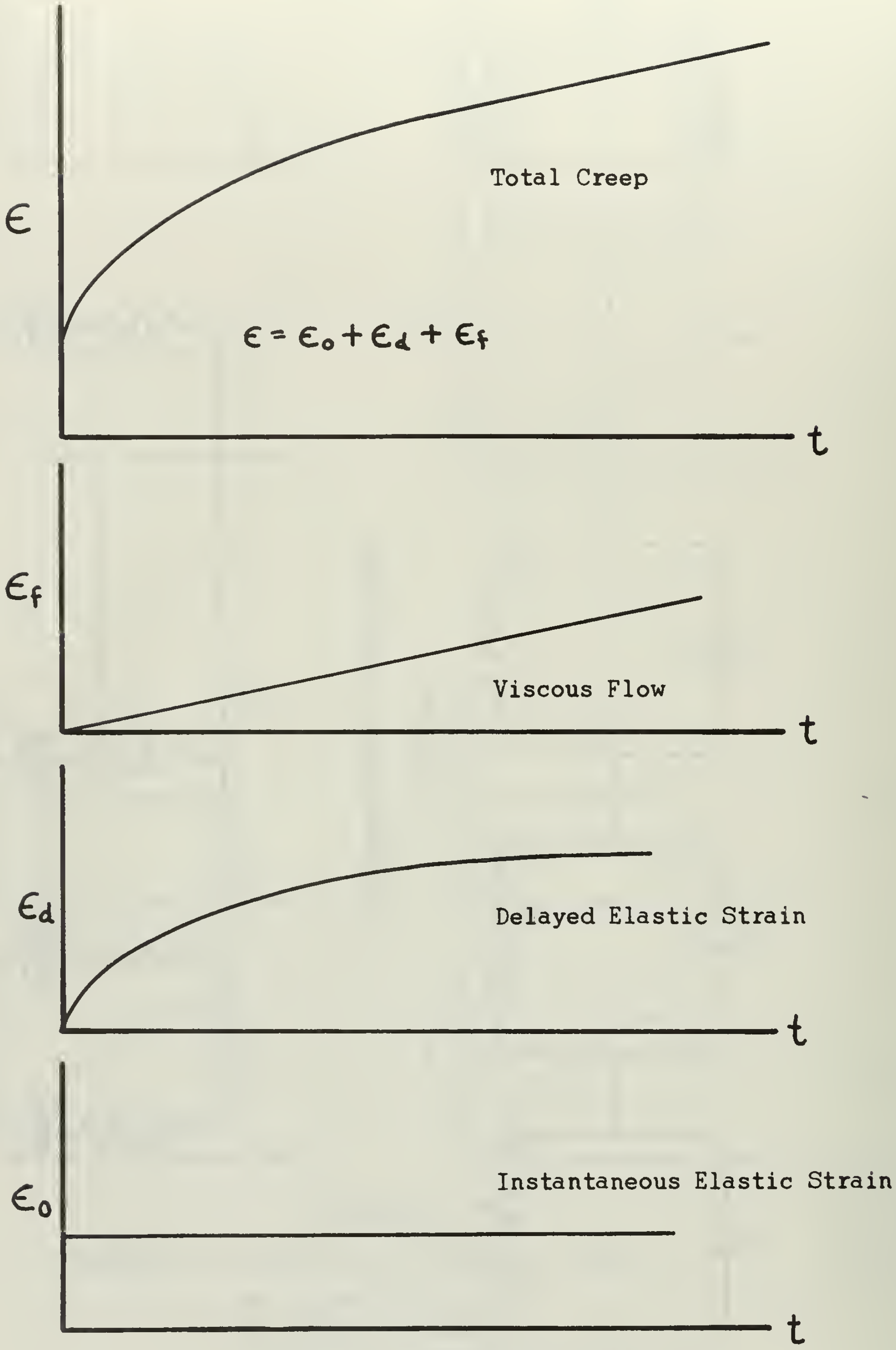

Figure 9

Creep Behavior of a Viscoelastic Material

$$\epsilon = \frac{\sigma}{E_2} + \frac{\sigma}{E_1}\left(1 - e^{-\frac{E_1}{\eta_1}\cdot t}\right) + \frac{\sigma}{\eta_2}\cdot t$$

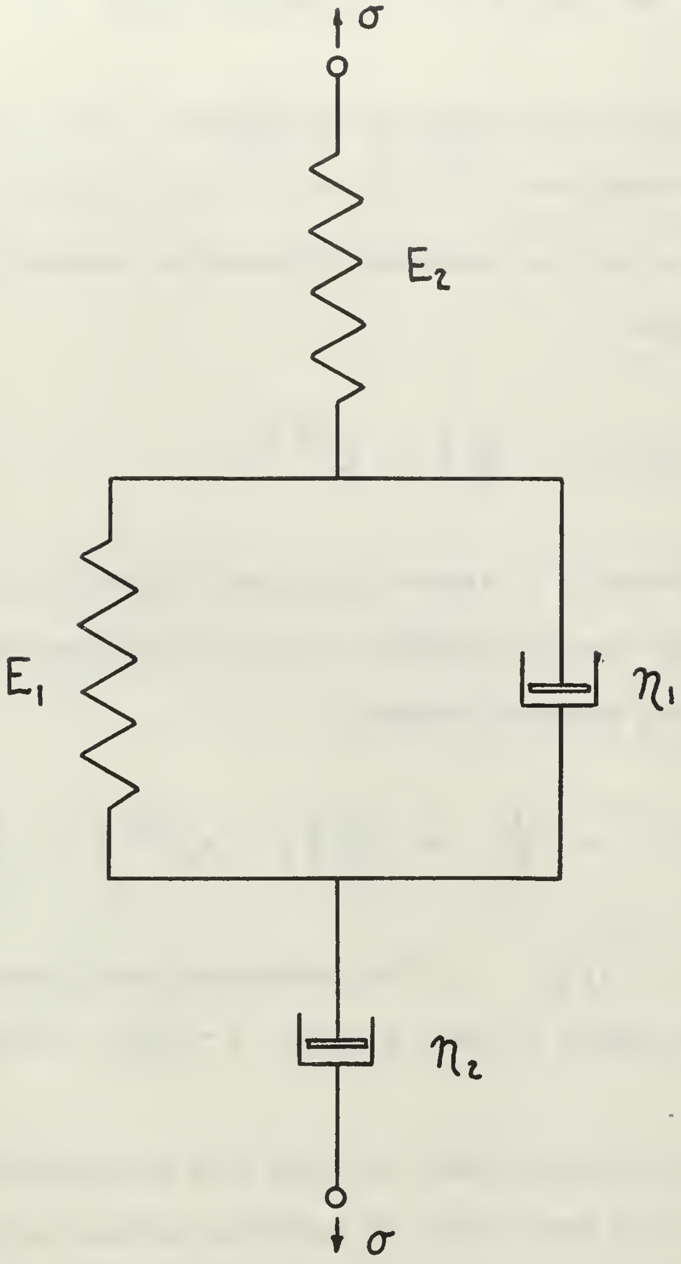

Figure 10

The Four-Element Model For Creep Behavior to the establishment of elastic equilibrium. [10] A corresponding mechanism in a polymeric solid might be the uncoiling of long polymer chains from their minimum energy unstressed positions, which involves diffusion of chain segments and takes considerable time. From the equations representing the components of the Voigt element,

$$(3.1) \qquad \sigma = E\epsilon_d + \eta \frac{d\epsilon_d}{dt}$$

The solution to this governing differential equation gives the strain-time relation:

$$(3.2) \qquad \epsilon_d = \frac{\sigma}{E}\left(1 - e^{-\frac{E}{\eta}\cdot t}\right)$$

The creep behavior of linear viscoelastic materials illustrated in Figure 9 may then be described by the four-element model of Figure 10. The resulting equation becomes:

$$(3.3) \qquad \epsilon = \frac{\sigma}{E_2} + \frac{\sigma}{E_1}\left(1 - e^{-t/\tau}\right) + \frac{\sigma}{\eta_2}\cdot t$$

where $\tau = \eta_1/E_1$ is the retardation time, the time required for the delayed strain to reach a value $1 - 1/e$, or 63.2% of its final value.

The four-element model has been used successfully by Gearhart and Kennedy [17] in their study of cellulose acetate butyrate plastics, and by Dienes [18] to correlate data on creep of polyethylene at elevated temperatures. Marin and Pao [19] have extended the four parameter equation to account for non-linear stress behavior frequently encountered in the delayed strain and viscous flow components for a simple plastic. Their equation (in its simplest form) can be written:

$$\epsilon = \frac{\sigma}{E} + C_1 \sigma^{m_1}\left(1 - e^{-C_2 t}\right) + C_3 \sigma^{m_2} \cdot t \tag{3.4}$$

A modified version of this equation was developed to characterize the creep behavior of the materials studied in this investigation.

4. Non-Linear Viscoelastic Behavior.

In a previous study [6] involving polymeric materials in the same loading ranges used in this investigation, marked non-linear behavior was encountered in the elastic, delayed elastic, and flow regions of stress response. Such deviations from linear viscoelasticity are not uncommon. [20] In a study of creep in cellulose acetate, Haward [21] found that the delayed elastic strain component increased by a factor of six when the stress was increased 30%. This non-linear behavior occurred at a comparatively low stress of 500 psi. Similar stress non-linearities have been observed for glass-like amorphous polymers [22], polymers in the transition region just below T_g [2], and polymers in the rubbery state [23]. In all cases where deviations from linearity occur, an increase in stress produces a more-than-proportionate increase in strain.

Alfrey [10] demonstrated that for large values of strain, non-linearity can be expected. From the theory of elasticity the work required to distort a solid to a given strain, called the "free energy of deformation," can be represented by a power series:

$$\mathcal{F}(\gamma) = c_2\gamma^2 + c_3\gamma^3 + c_4\gamma^4 \ldots\ldots \qquad (4.1)$$

The shear modulus G has a magnitude given by:

$$G = \frac{d^2\mathcal{F}}{d\gamma^2} \qquad (4.2)$$

For small values of strain only the quadratic term is significant and the modulus is a constant, independent of strain γ. At larger strains, when higher order terms become important, the shear modulus becomes a function of γ, and stress non-linearities result. A similar analysis predicts non-Newtonian flow of viscoelastic materials at higher stresses.

For glassy polymers which have high moduli and can support large stresses, non-linear behavior has been observed even when the strains are relatively small. [2] This suggests that internal structural changes are responsible for the deviations. For example, it is reasonable to suppose that the flow rate will be markedly different for polymer chains which are straight than for a coiled chain. Experimental measurements in this field are sparse, but some theoretical aspects have been discussed by Schwarzl in terms of transition-state kinetics. [24]

Nonlinearity of viscous and elastic elements of a model create systems of nonlinear differential equations which are difficult to solve by ordinary methods. In this investigation an equation similar to that of Marin and Pao (Equation 3.4) was used to fit creep data for various polymers. By making some simplifying assumptions in the equation, and with the aid of a digital computer, satisfactory predictions of composite behavior were obtained.

5. The Experiment.

The viscoelastic materials selected for the investigation were: high density linear polyethylene, isotactic polypropylene, nylon 6/10, and polycarbonate. All were obtained from commercial sources in the form of extruded sheet. General mechanical properties of these plastics determined by tension testing, along with approximate values of other physical constants from the literature [15, 25] are tabulated in Appendix A. Creep test specimens were cut and machined to the dimensions shown in Figure 11. The thickness of the specimens was that of the sheet material; approximately 0.060 inches for polycarbonate and nylon, and 0.192 inches for polypropylene and polyethylene. Specimens used for relaxation tests and tensile tests were made with the dimensions shown in Figure 12. All specimens were cut with the long axis parallel to the extrusion direction.

Creep tests were carried out on constant load creep machines in a laboratory with the temperature maintained at 23 ± 1° Celsius and relative humidity controlled at 50 ± 2%. Construction features of the machines and the method of gripping the specimen are discussed in Appendix G. Series of creep tests were run at different stress levels, ranging between 10 and 80 percent of the nominal yield strength. Extension of the test specimens over a two-inch gauge length was measured with a Tripolitis type extensometer which could be read to ± 0.0001 inches. After loading the specimens, readings were taken at elapsed times of 6, 12, 30, 60, 90, 120 seconds, and then in increasing intervals from 1 to 10 minutes during the first hour, then every 30 minutes during the first day. The test was continued for 100 to 150 hours, with an average of 50 data points taken during the run. Similar test procedures were followed for the composites, with the component materials clamped together outside the

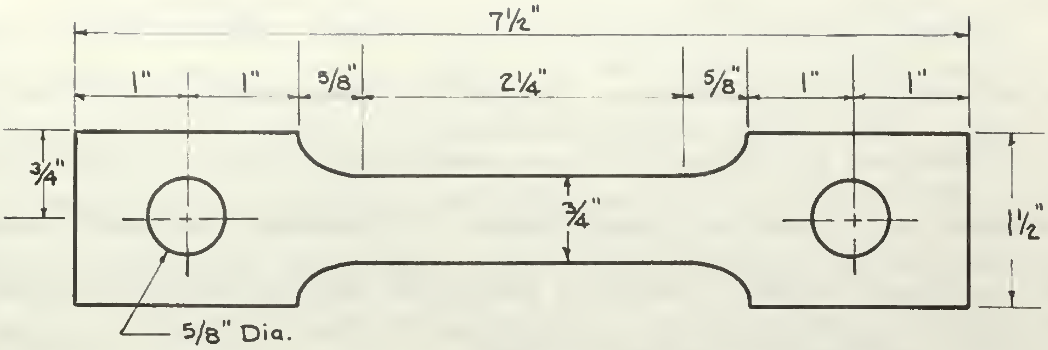

Figure 11

Creep Test Specimen

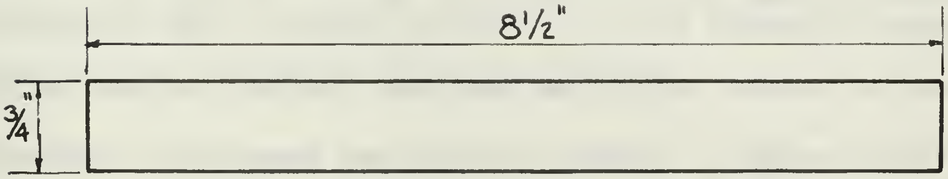

Figure 12

Tensile Test Specimen

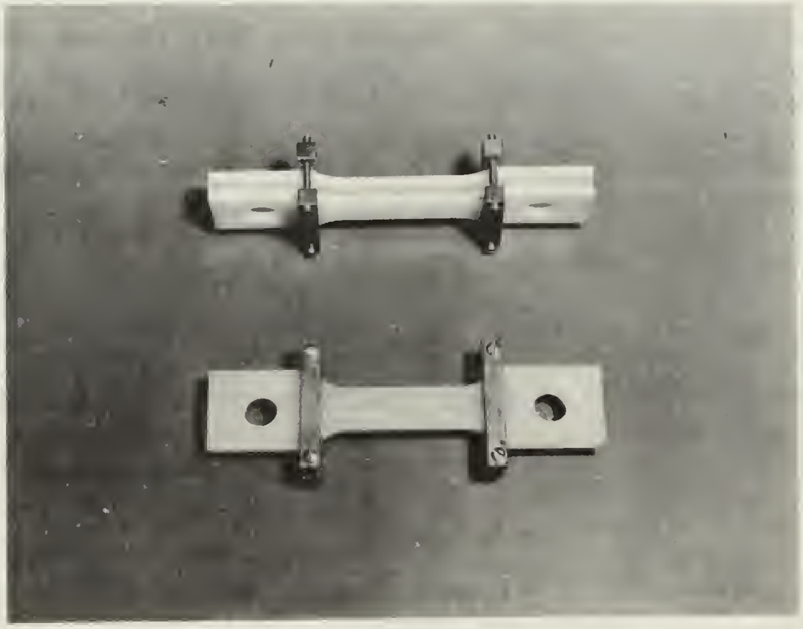

Figure 13

Clamped Composite

gauge length to insure uniform elongation between the clamps. (See Figure 13)

Uniaxial tension tests were run at the same temperature and humidity on an Instron testing machine. Specimens were pulled at a constant rate of deformation, and Tripolitis extensometers were used to measure the extension over a two-inch gauge length. Preliminary experiments measuring the strain at timed intervals indicated that the strain rate was essentially constant throughout the test.

Stress relaxation tests were performed on the Instron tester in the following manner. Samples were strained rapidly at ten inches per minute to a predetermined load level, where a load limit microswitch automatically stopped crosshead motion. Graphical output of load relaxation versus time at constant strain was obtained from the recorder installed on the testing machine. Initial straining was essentially instantaneous; less than one second was required to reach the desired stress. Duration of the relaxation tests was approximately thirty hours.

A portion of the creep data for polypropylene and polyethylene was obtained in a previous study. Because this work [6] was done in the same laboratory utilizing identical test procedures and specimens cut from the same stock, no distinction is made in the data tabulation in Appendix A.

6. Selection of an Equation to Characterize Creep Behavior.

The four-element model (Figure 10) discussed in Section 3 describes the creep process in three stages; an instantaneous elastic strain, a delayed elastic response, and non-recoverable plastic flow. While there is considerable theoretical justification for this description of creep behavior [11, 27], this simple form of the model has two serious shortcomings. First, it fails to account for the non-linear phenomena frequently encountered in tests made at stresses above 25% of the yield strength. Improved fits with actual data can be achieved by modifying the basic equation (Equation (3.3)) to account for these stress anomalies. Such a modification is shown in equation (3.4), and this method was utilized for data fitting in this investigation.

$$\epsilon = \frac{\sigma}{E_2} + \frac{\sigma}{E_1}\left(1 - e^{-t/\tau}\right) + \frac{\sigma}{\eta_2} \cdot t \qquad (3.3)$$

$$\epsilon = \frac{\sigma}{E} + c_1 \sigma^{n_1}\left(1 - e^{-c_2 t}\right) + c_3 \sigma^{n_2} \cdot t \qquad (3.4)$$

The second limitation imposed by the four-element model is more fundamental. The presence of only one Voigt Element implies that the delayed elastic response of a polymer is a single molecular mechanism with one characteristic retardation time. As pointed out by Alfrey [15], the coils and linkages of a polymer chain have a multitude of sizes and spatial arrangements. One would expect local kinks to be straightened out more rapidly than long range coils, and that elastic equilibrium would be achieved sooner if the chain segment had a favorable orientation with the stress axis. The delayed elastic response of a viscoelastic material would then consist of several mechanisms with different retardation times.

The "memory" effects observed in high polymers are frequently cited as proof of the existence of more than one retardation time. Failure of a single Voigt Element to represent accurately the delayed elastic response in the materials used in this study is discussed in Appendix F.

Figure 14 illustrates schematically the shape of a creep curve for a viscoelastic substance compared to a set of simple exponential curves of the form $\epsilon = K(1 - e^{-t/\tau})$, and shows the failure of a single time constant concept to represent actual data. Figure 15 shows the retardation spectra corresponding to the simple exponentials. In this figure, the term b represents the rate constant (= $1/\tau$) and $g(b)$ the rate constant distribution function. The creep curve of Figure 14 can be approximated very closely by a finite number of simple exponentials appropriately weighted. The model representing the delayed elastic response would then be the generalized Voigt Model of Figure 7. Such a representation consisting of exponentials ①, ②, and ③ is given in Figure 16. A generalized equation for n discreet relaxation times, with the instantaneous strain (A_1) and the flow strain (A_2 x t) included would be:

$$(6.1) \qquad \epsilon = A_1 + A_2 \cdot t + A_3\left(1 - \sum_{i=1}^{n} a_i e^{-b_i t}\right)$$

where $a_i = g(b_i)$, the discontinuous distribution function for the $i^{\underline{th}}$ exponential, and $\sum a_i = 1$. Although this "spike distribution" model was not used in this study to correlate creep data, G. F. Kinney, in a parallel investigation at NPGS, has used the model to fit the same data. In his work, rate constants were arbitrarily assigned, then appropriately weighted using a least squares curve fitting technique adapted to a digital computer. Two to six retardation times were used, and excellent fits were obtained, even with the simple two-spike distribution.

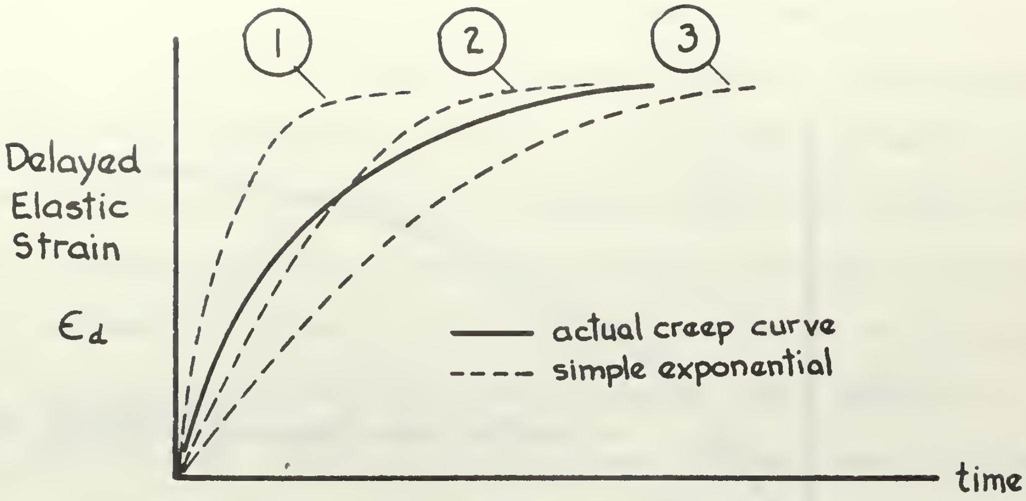

Figure 14

Comparison Of Creep Curve With Simple Exponential Curves

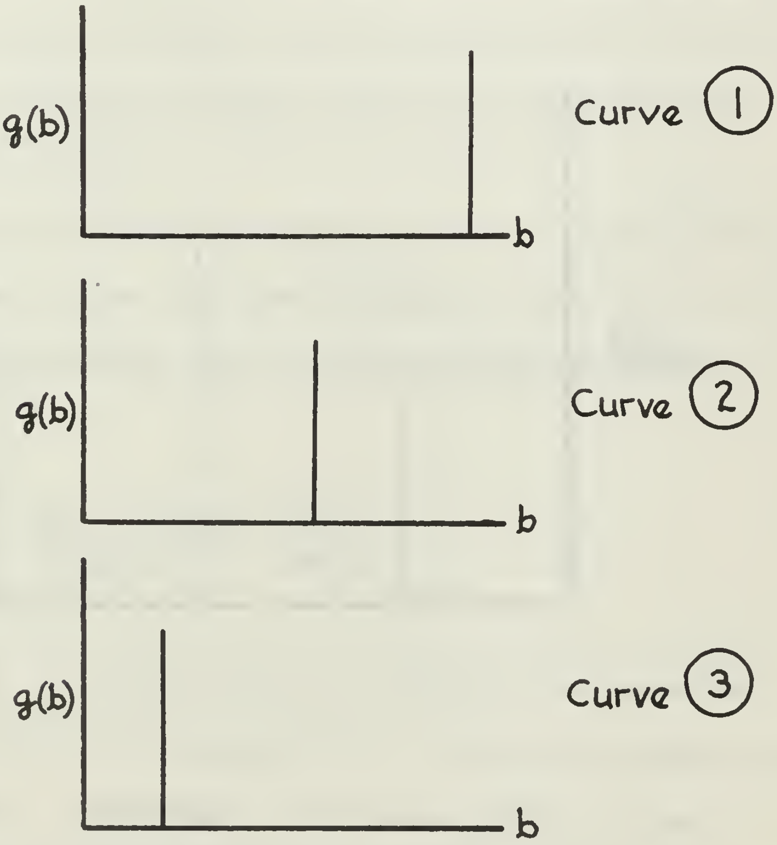

Figure 15

Retardation Spectra Corresponding to the Simple Exponential Curves of Fig. 14

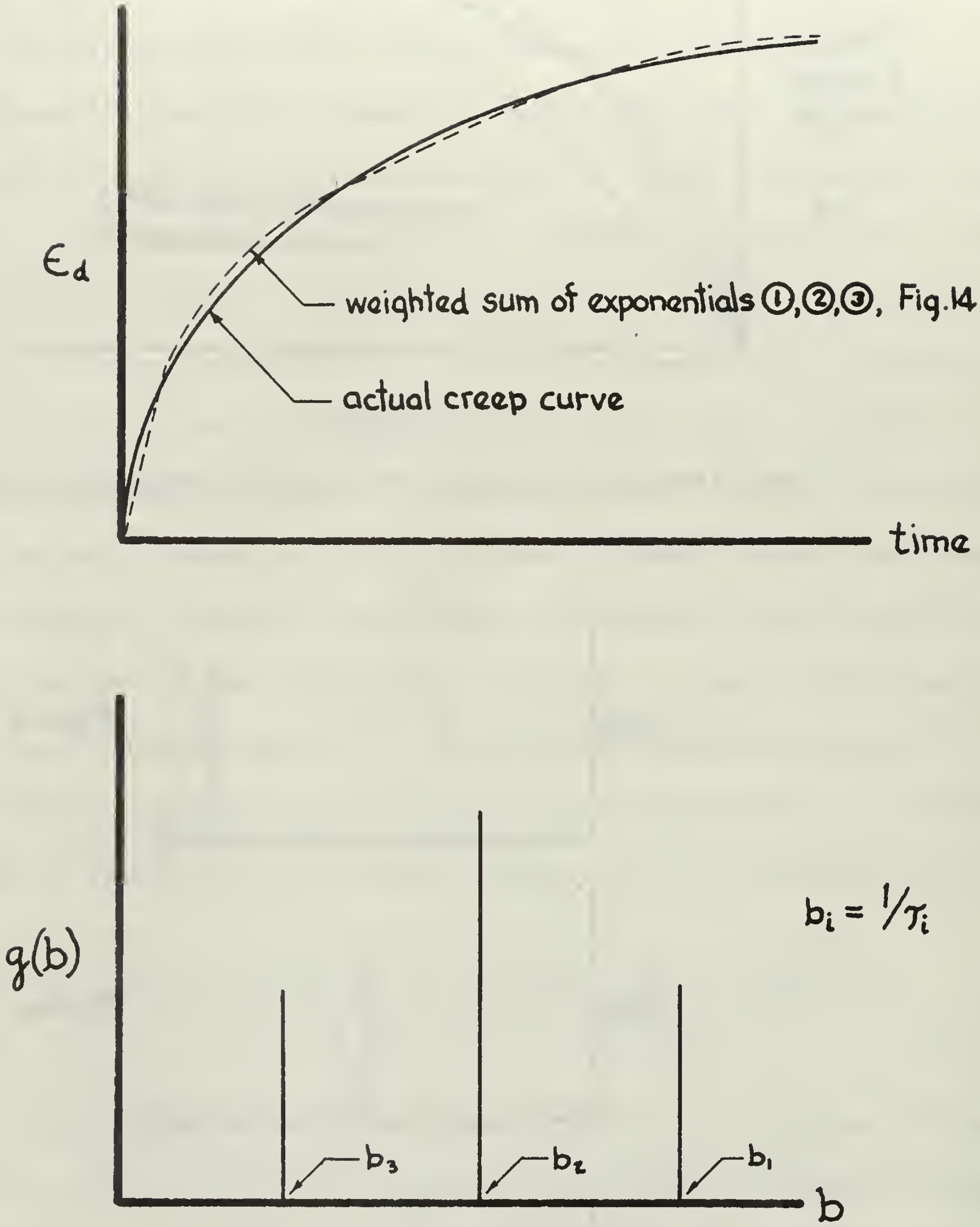

Figure 16

Representation of Creep Behavior By Spike Distribution of Retardation Times

Another method of representing delayed elastic behavior is by a continuous distribution of retardation times, $g(b)$. This concept is attractive from a theoretical standpoint if one considers delayed elasticity as consisting of many molecular mechanisms whose rate constants form a quasi-continuous spectrum. Alfrey [10] develops a molecular theory of delayed response and arrives at a theoretical distribution of retardation times for a monodisperse amorphous linear polymer. This distribution, whose upper and lower limits can be determined from the theory, is shown in Figure 17. (The notation here is left in the form given by Alfrey) For a polymer consisting of two different molecular weights, Alfrey has deduced the retardation spectrum shown in Figure 18. Tobolsky, Andrews, and Dunnel [28] used a box distribution (Figure 19), to correlate stress relaxation data. The limits of such a box distribution may be obtained graphically as shown in Figure 19 from a data plot of delayed elastic strain versus logarithm of time.

The calculation of the retardation spectrum for an actual material, where the time constants extend over many decades of time, can be an involved process. A simple method [15] of approximating the spectrum is given by:

$$L(\tau) = \frac{d}{d\ln\tau}\left(J(t) - t/\eta\right) \tag{6.2}$$

Here,[$J(t) - t/\eta$] corresponds to the delayed strain response, $\epsilon_d(t)$, and $L(\tau)$ is the distribution of retardation times which can be converted to the rate constant distribution $g(b)$. Figure 20 demonstrates this graphically by plotting the slope of the delayed strain vs. logtime curve versus logtime. It is apparent that if Figure 20 does represent the response of the four-element model of Figure 10, the approximation gives a curve with a peak at the most probable τ instead of a single

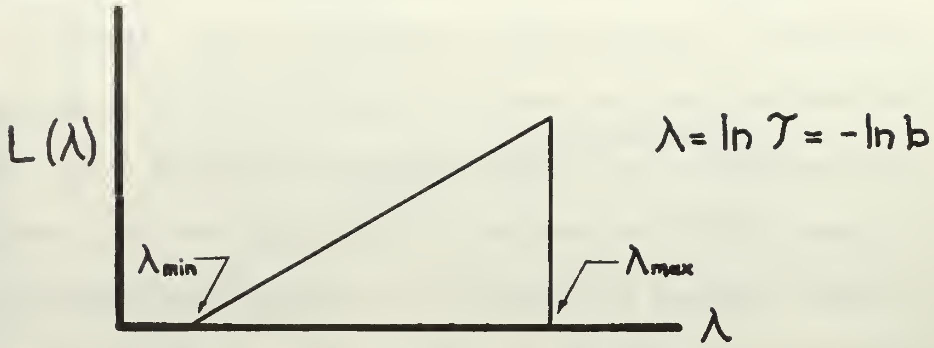

Figure 17

Retardation Spectrum for a Monodisperse Amorphous Linear Polymer (Ref. [10])

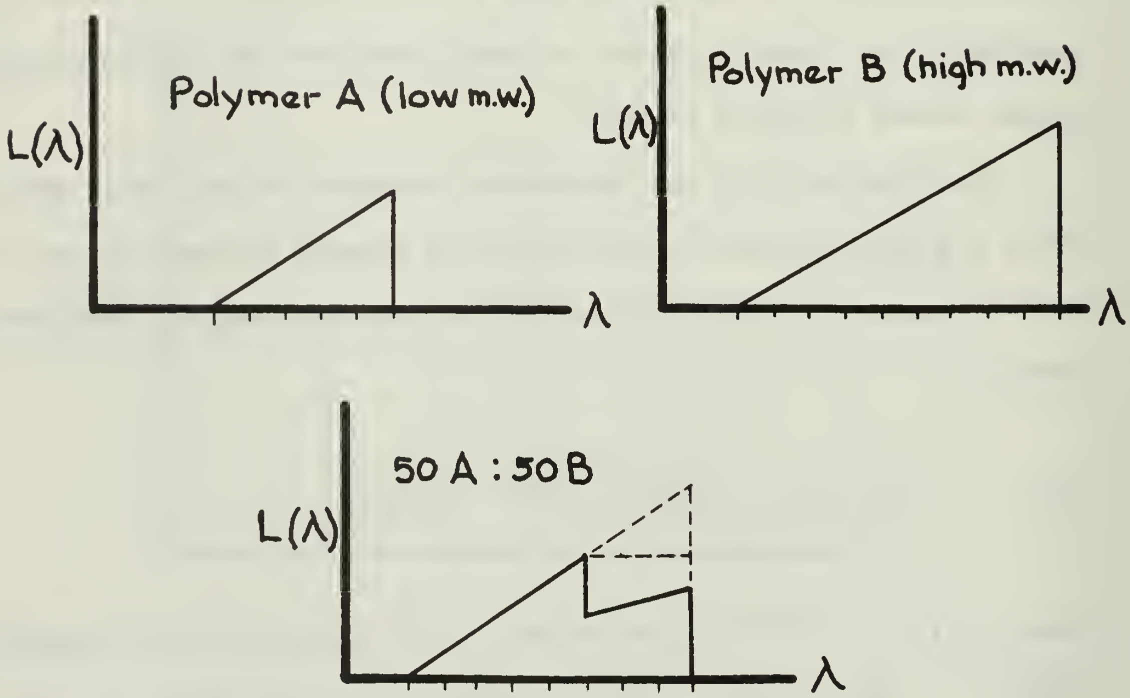

Figure 18

Spectrum for a Polymer Consisting of Two Different Molecular Weight Fractions

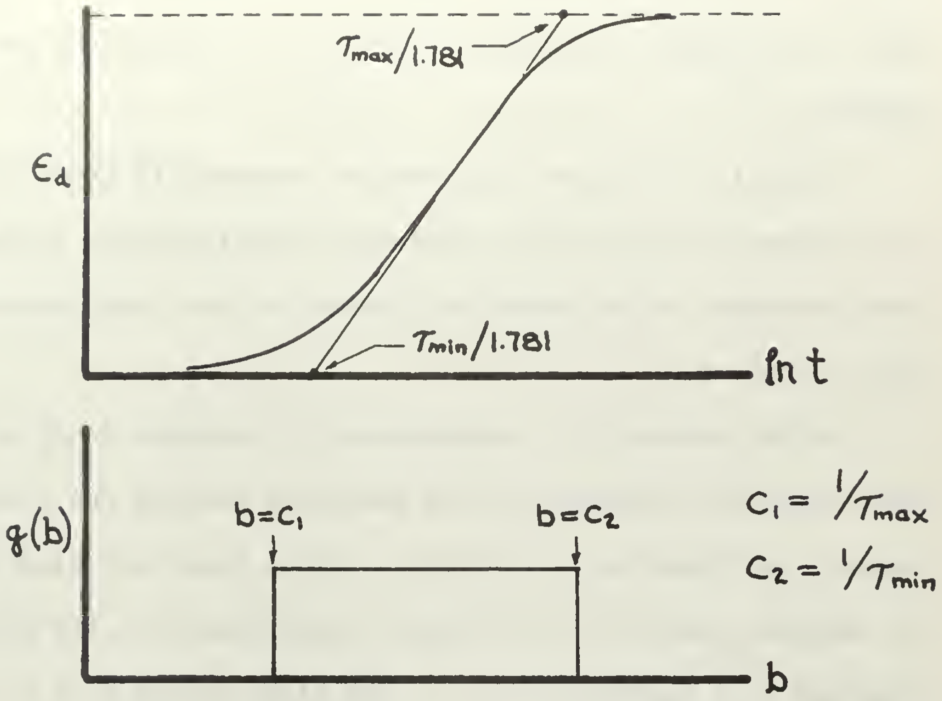

Figure 19

Box Distribution Of Retardation Times

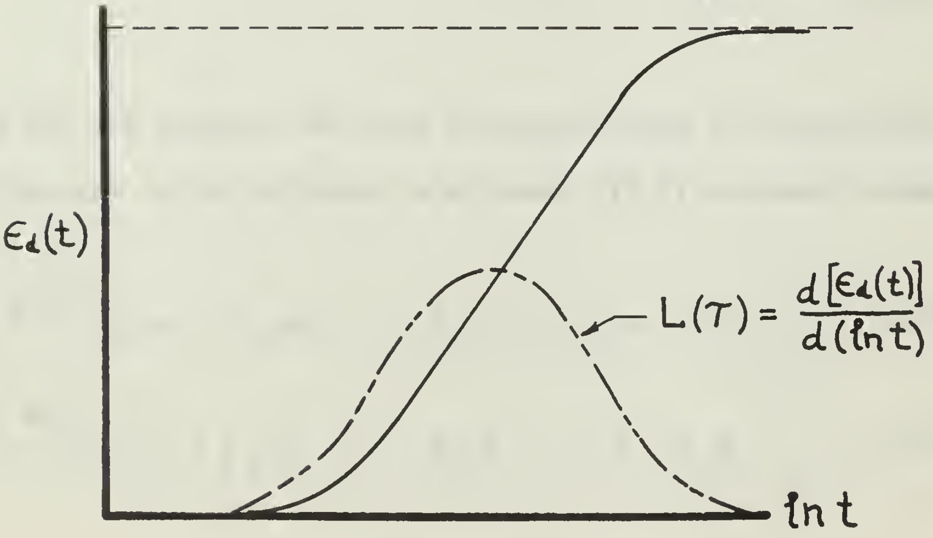

Figure 20

Method Of Approximating Retardation Spectra spike. For real materials, however, the approximation is better. Reference [27] reviews more accurate methods for estimating the retardation spectra.

Analysis of the box distribution function is given in Appendix F. It is shown that this distribution is unsuitable for characterizing the creep behavior of the materials tested, as are other monomodal distribution functions.

In the absence of a mathematically tractable function to represent the retardation spectrum of the materials tested, the continuous spectrum concept was discarded. Instead, a simple empirical equation was developed by assuming that the rate constant associated with the delayed elastic response is a function of time. One might expect that a polymer chain segment should exhibit a changing retardation time as the uncoiling process continues and the shape of the molecule changes. Thus, an overall characteristic time constant τ_R might be considered, with a simple power dependency on time:

$$\tau_R = a t^n \tag{6.3}$$

Substitution of this expression into the equation for the four element model (Equation (6.4)) leads to an equation of the form of equation (6.5)

$$\epsilon = A_1 + A_2 t + A_3 \left(1 - e^{-t/\tau_R}\right) \tag{6.4}$$

$$\epsilon = A_1 + A_2 t + A_3 \left(1 - e^{-ct^Q}\right) \tag{6.5}$$

Equation (6.5) was used to fit creep data in this investigation. The method of obtaining the creep parameters A_1, A_2, A_3, C, and Q are discussed in the next section.

7. Analysis of the Data.

Creep data were taken for all specimens in the form of elongation versus time. These data were punched on cards and a print-out obtained from an IBM 360 Digital Computer. The printout served as a check to pick up gross errors due to equipment malfunction during the testing period or errors made in transcription of the data.

The first step in fitting the creep data was to obtain the time-independent part of the elastic strain. This instantaneous elastic strain, ϵ_0 , occurring when the load is applied, is represented by the parameter A_1 in equation (7.1):

$$\epsilon = A_1 + A_2 t + A_3(1-e^{-ct^Q}) \tag{7.1}$$

To evaluate the constant A_1 , a direct method was employed. The first four data points were used to perform an extrapolation back to zero time. This extrapolation to determine the parameter A_1 is illustrated in Figure 21. It is clear from this figure that this evaluation of A_1 is at best a rough estimate due to the nonlinear nature of the extrapolation. It will be shown in Appendix D that the parameter A_1 does include a portion of the delayed elastic strain, but for the time frame of this experiment, the strain A_1 occurs essentially instantaneously.

To account for the non-linear viscoelastic behavior encountered, the parameters A_1 , A_2 , and A_3 were correlated with stress so that the creep behavior could be predicted for any stress level. The generalized form of equation (7.1) becomes:

$$\epsilon = K\sigma^N + B\sigma^R \cdot t + P\sigma^M(1-e^{-ct^Q}) \tag{7.2}$$

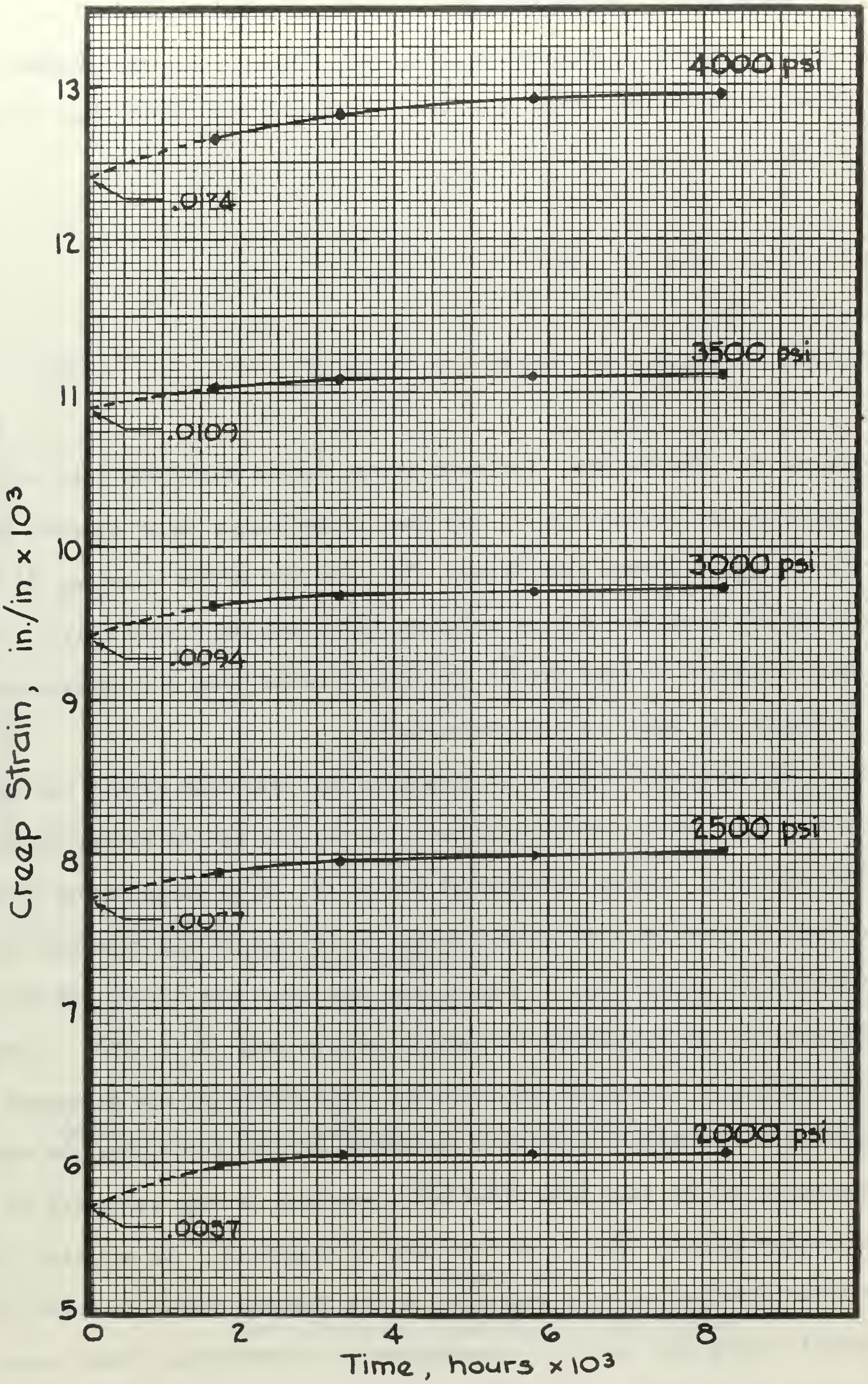

Figure 21

Extrapolation of Creep Data to Determine Instantaneous Strain

The magnitude of parameters A_2 and A_3 depend on the value chosen for A_1. Prior to evaluating A_2 and A_3, the extrapolated values of zero time strain A_1 were correlated with the first term of equation (7.2):

$$A_1 = K\sigma^N \tag{7.3}$$

$$\ln A_1 = \ln K + N \ln \sigma \tag{7.4}$$

Values of $\ln A_1$ for different stress levels were read into a digital computer, and equation (7.4) was used to perform a least squares fit to determine the stress coefficient K and the stress exponent N. Figure 22 shows the plot of equation (7.4) and the fitted curve. The data displayed in this figure are that obtained from the extrapolations of Figure 21, made for polycarbonate.

An independent method of obtaining the zero-time strain was attempted by running a series of tensile tests at various strain rates, and plotting the resulting data as sketched in Figure 23. From these stress-strain curves, a cross-plot of strain versus reciprocal strain rate may be constructed as illustrated in Figure 24. The zero-time strain can then be obtained by extrapolating the data to strain rate $\dot{\epsilon} = \infty$, which corresponds to instantaneous loading. This technique was attempted using a digital computer to perform a parabolic extrapolation, but the exponential character of the data plot invariably resulted in high estimates of the zero-time strain, ϵ_0, as indicated in Figure 24. In addition, the maximum deformation rate available on the Instron tester used for the tensile tests was $\dot{\delta}$ = 10 inches/minute. Consequently, there was a lack of data in the region of greatest interest, i.e., where $1/\dot{\epsilon} \rightarrow 0$. This

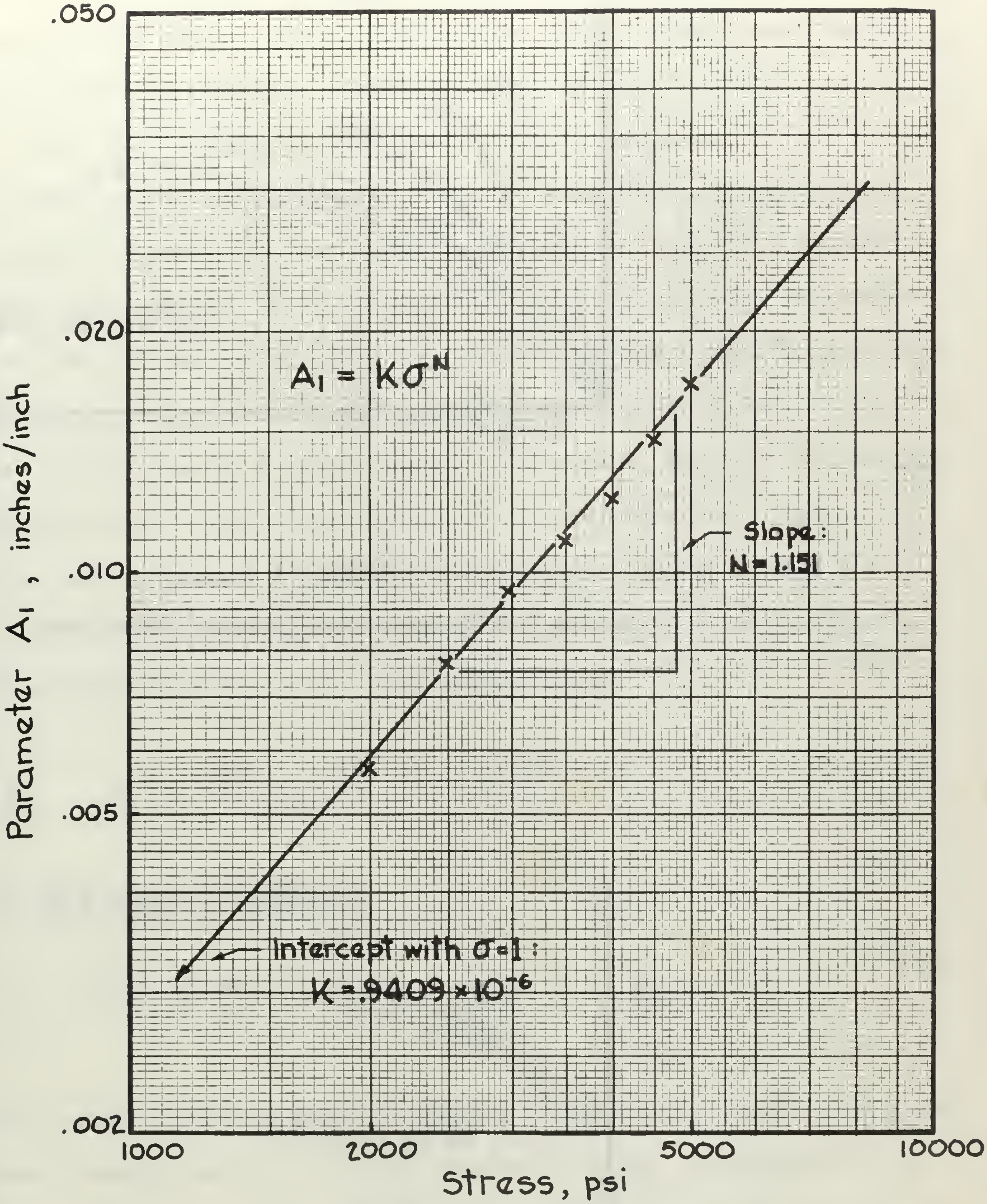

Figure 22

Correlation of Parameter A_1 with Stress

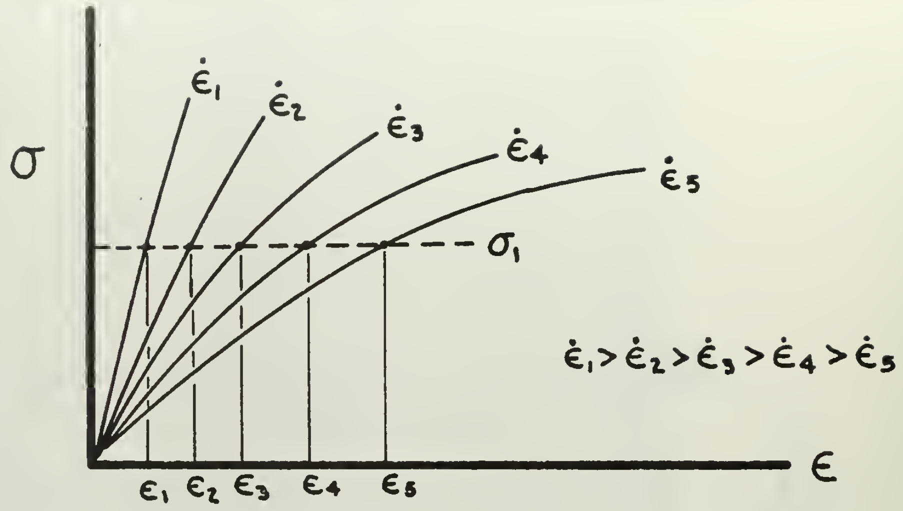

Figure 23

Stress-Strain Curves for Various Strain Rates

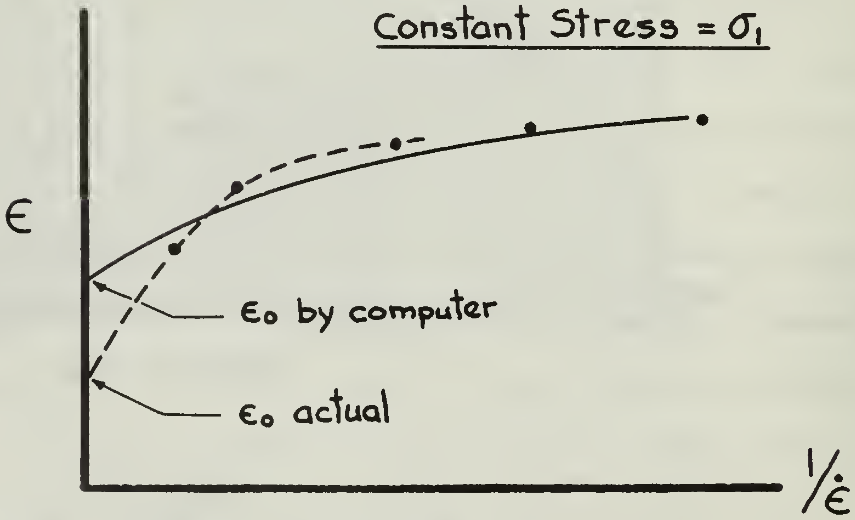

Figure 24

Zero Time Strain Extrapolations from Stress-Strain Data method may be expected to produce excellent results if high-speed testing apparatus were available [29], but in view of the limitations discussed above it was discarded in favor of the creep data extrapolation technique.

After the parameter A_1 was found by extrapolation and "smoothed" by the stress correlation (Equation (7.3)) the creep data, along with the calculated parameter A_1, was placed in the digital computer via program FITDATA (See Appendix B). The viscous flow parameter A_2 was determined by a linear extrapolation using data points at elapsed times greater than 70 hours. It was assumed that elastic equilibrium was achieved by this time. With parameter A_1 specified and parameter A_2 computed, the maximum magnitude of the delayed elastic strain, A_3, is determined. This procedure used by program FITDATA is outlined in the sketch of Figure 25. The time constant parameters were computed by FITDATA by evaluating the delayed strain fraction, $\mathcal{E}$, at various times.

$$\mathcal{E} = \epsilon_d / A_3 = 1 - e^{-ct^Q} \tag{7.5}$$

from which

$$-\ln(1-\mathcal{E}) = ct^Q$$

$$\ln\left[-\ln(1-\mathcal{E})\right] = \ln C + Q \ln t \tag{7.6}$$

A plot of the left-hand side of equation (7.6) versus $\ln t$ was given a least squares curve fit, and the constants C and Q determined. The method is sketched in Figure 26, and graphical computer output of actual data is displayed in Appendix B.

The creep parameters A_2 and A_3 were correlated with stress using equations (7.7) and (7.8):

$$A_2 = B\sigma^R \tag{7.7}$$

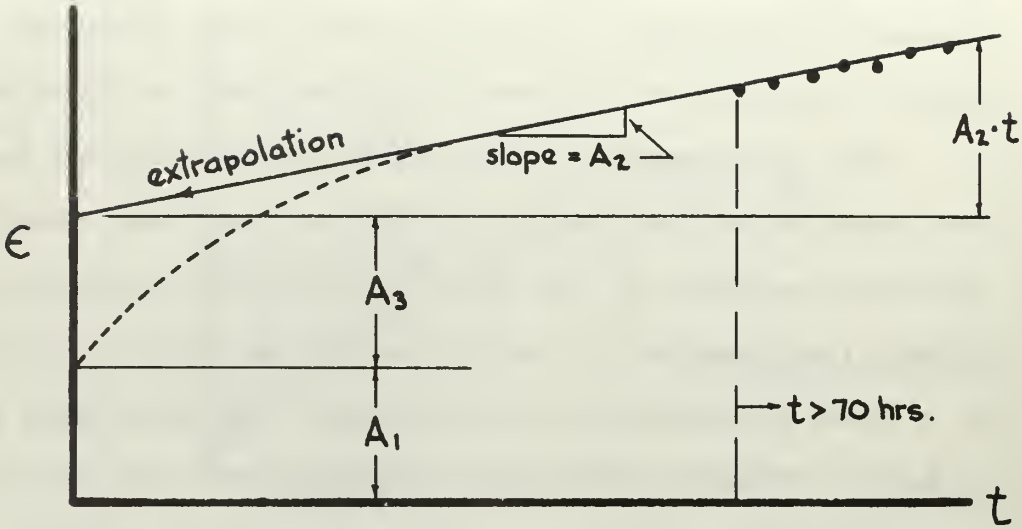

Figure 25

Determination of Parameters A_2 and A_3

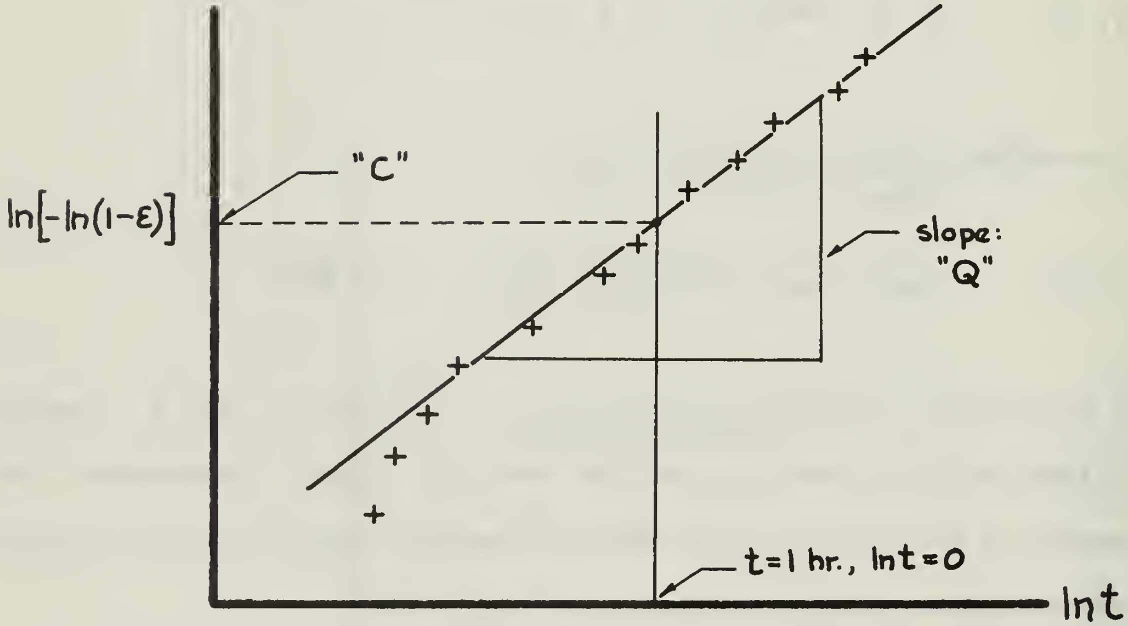

Figure 26

Determination of Parameters C and Q

(7.8) $$A_3 = P\sigma^M$$

The method used for the A_1 correlation was employed to determine the stress exponents and coefficients in equations (7.7) and (7.8). These determinations for polycarbonate are illustrated in Figures 27 and 28.

A question arises as to the validity of inclusion of the viscous flow term $A_2 \cdot t$ in equation (7.1). If the term is not included and the first and second derivatives of strain with respect to log time are taken, at long times the second derivative is always negative and becomes vanishingly small. At long times the first derivative, corresponding to the slope of the ϵ vs. $\ln t$ curve of Figure 29, approaches zero. With no viscous flow, elastic equilibrium is eventually reached and extension ceases. If the flow term $A_2 \cdot t$ is included, the second derivative changes sign and becomes increasingly positive at long times. This behavior is observed as an inflection and an upswing in the plot of ϵ vs. $\ln t$, Figure 30. If viscous flow is a factor in creep behavior, therefore, a data plot of ϵ vs. $\ln t$ should show the inflection and upturn indicated by Figure 30. In all the materials investigated this behavior due to viscous flow was observed, as illustrated by the example of Figure 31, justifying inclusion of the term $A_2 \cdot t$ in equation (7.1).

Values of the creep parameters determined for the materials tested are summarized in Table 2 of Appendix A. The constants C and Q are apparently independent of stress and are of the same magnitude for all materials tested. Standard deviations in strain were acceptable for all runs. The errors in computed strain were less than 5% in the region where delayed elastic behavior is significant, except at elapsed times under two minutes. Sample computer output from FITDATA listing point by point comparison of observed and computed strain is given in Appendix B. For typical values of the constants $C = 0.8$, and $Q = 0.35$, the delayed

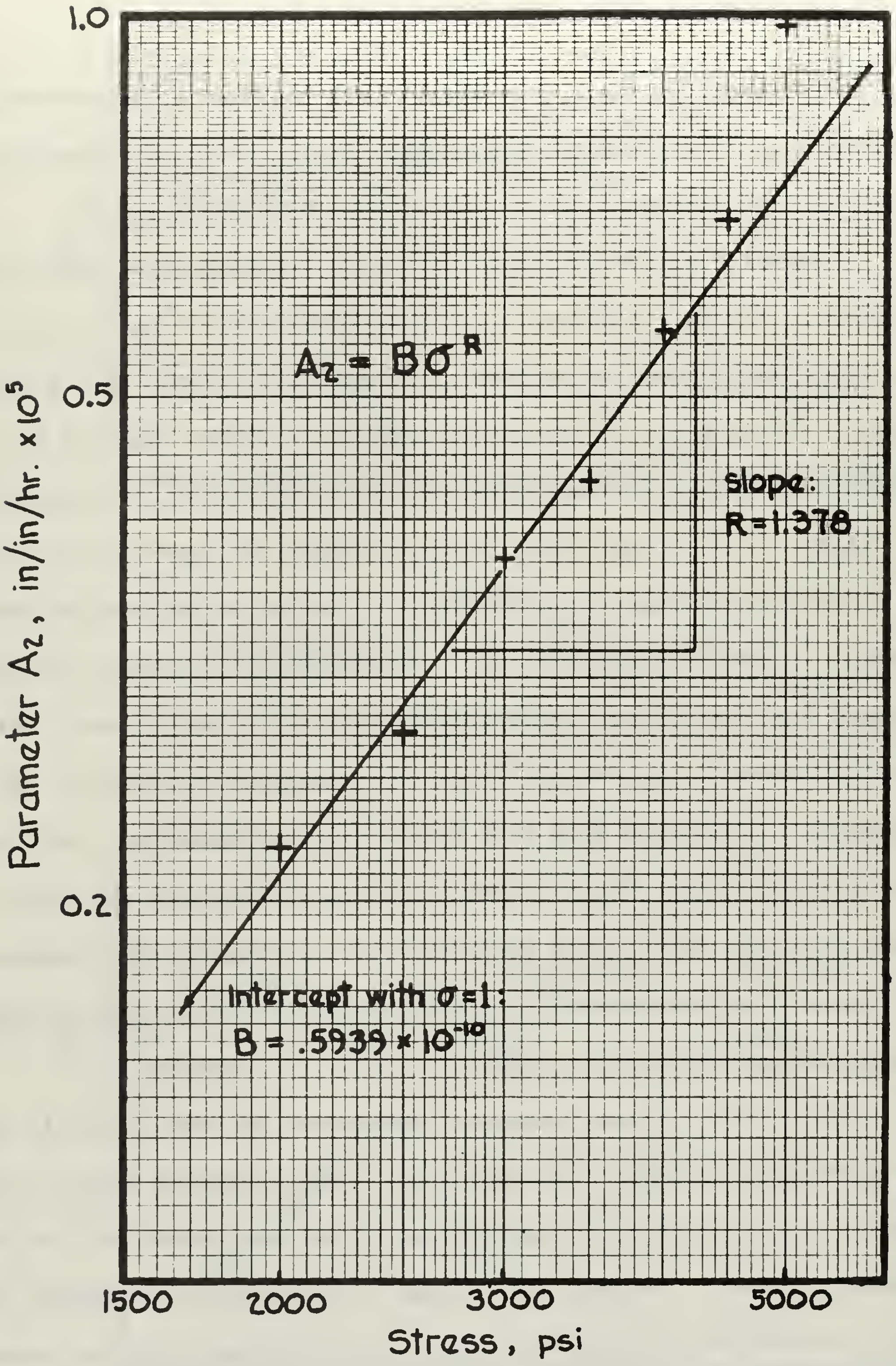

Figure 27

Correlation of Parameter A_2 with Stress

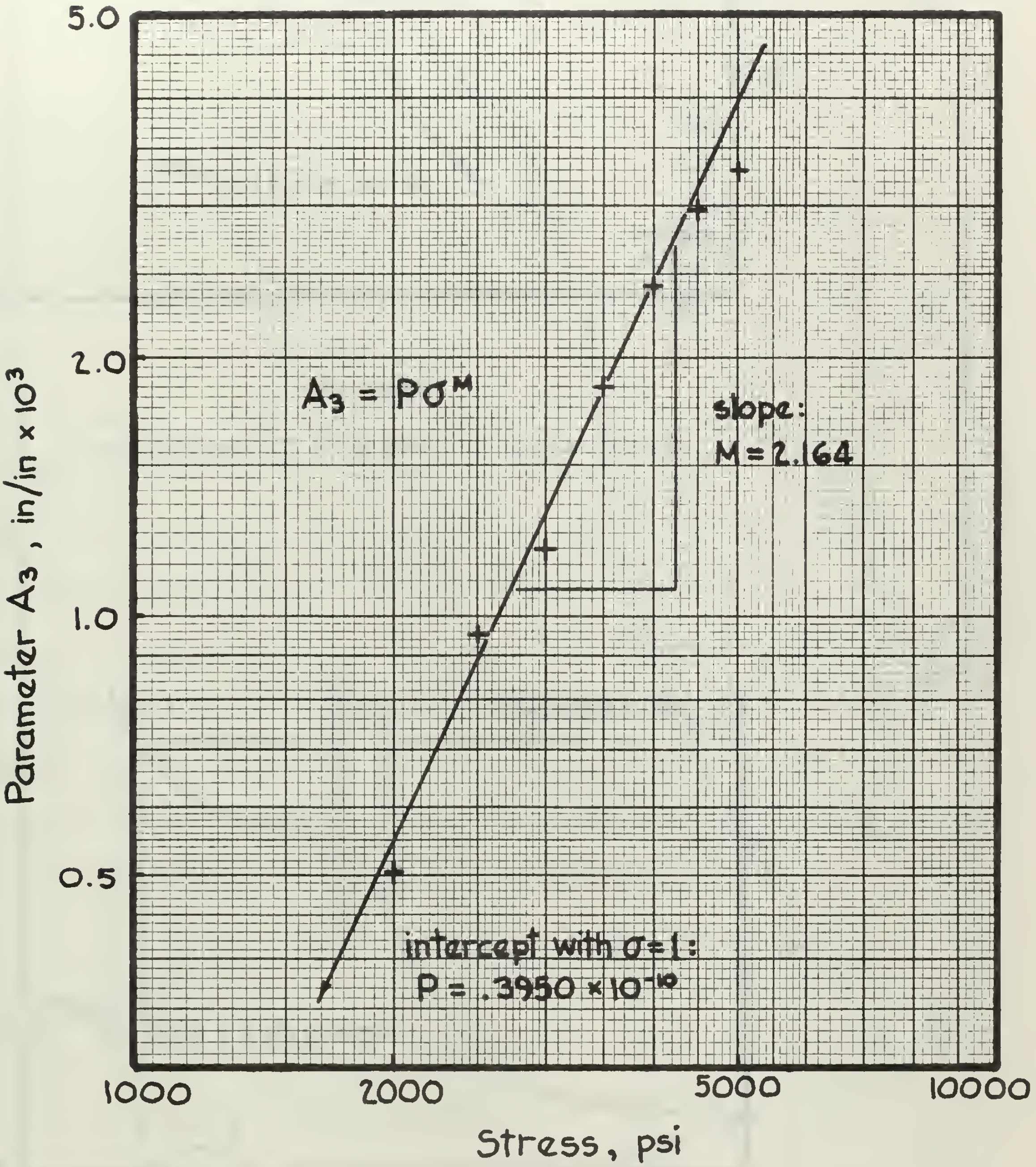

Figure 28

Correlation of Parameter A_3 with Stress

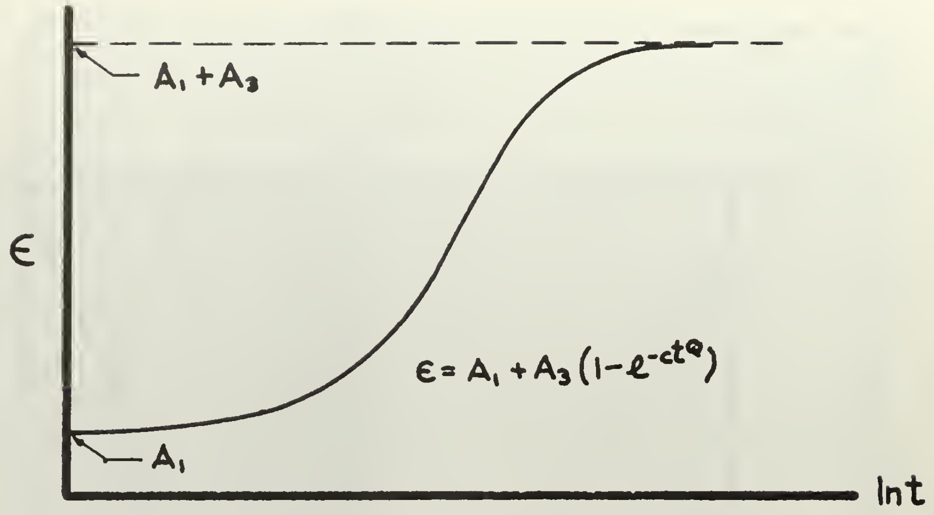

Figure 29

Strain versus Ln Time with No Viscous Flow

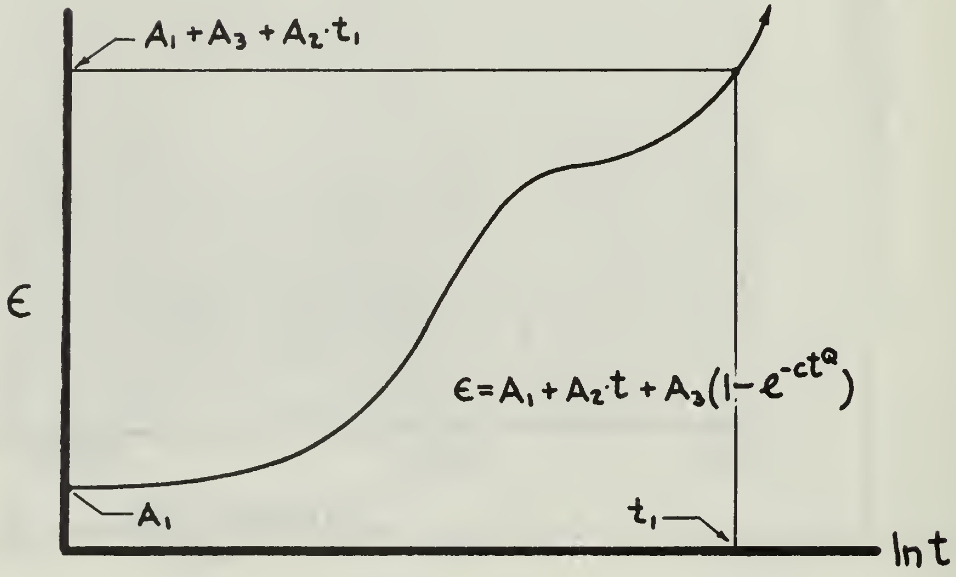

Figure 30

Strain versus Ln Time with Viscous Flow

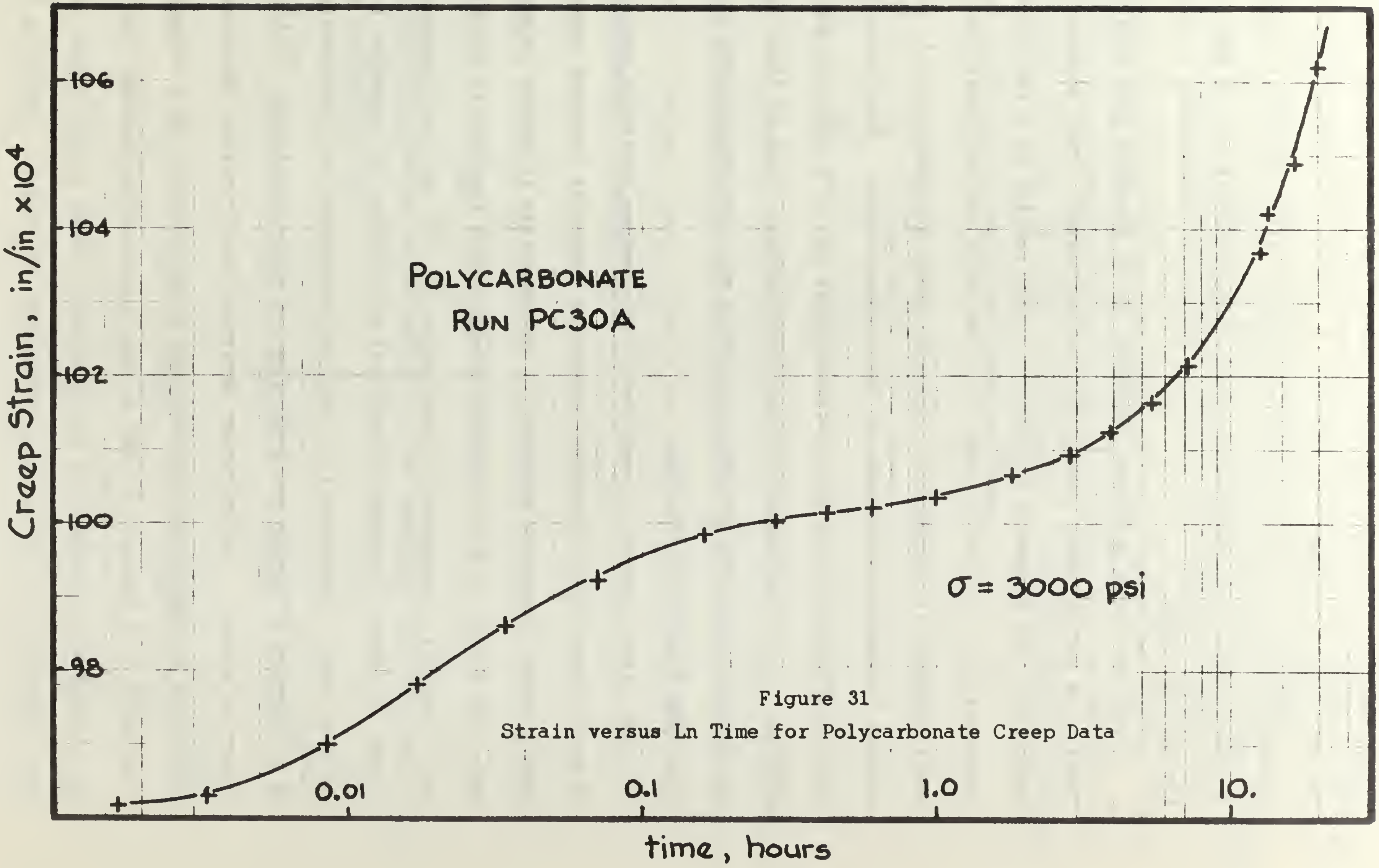

Figure 31

Strain versus Ln Time for Polycarbonate Creep Data strain is only 97% exhausted at an elapsed time of 70 hours. For the extrapolation to determine parameter A_2, elastic equilibrium is assumed. Consequently, in the resulting "forced fit," after the other parameters are determined, the computed strain is approximately one to two percent low at long times.

From the tests at higher stresses, it is apparent that elastic equilibrium is not achieved during the testing period. This produces a high estimate of the parameter A_2, which in turn results in a low estimate for parameter A_3. At low stresses the flow rates are very small and the error introduced is negligible. However, power dependence of A_2 on stress creates noticeable deviations at high stress levels. This deviation is clear in Figures 27 and 28 at stress levels of $\sigma = 4500$ psi and $\sigma = 5000$ psi. In performing the stress correlations, data at lower stresses was weighted more heavily, which corrects the erroneous values of A_2 and A_3. (See figures 27 and 28).

The fitted empirical equation (7.2) provides excellent predictions of creep behavior on individual materials loaded at constant stress. When a simple composite sandwich of two materials is tested in creep however, the stress in each component changes with time due to the different viscoelastic responses of each material. If the time responses were linear in stress, accurate theoretical predictions of composite behavior could be made. Reference [15] outlines procedures for calculating stress as a function of strain and time when the creep compliance $J(t)$ is linear in stress. The Boltzmann Superposition Principle (Equation (2.11)) could then be applied to determine the creep behavior of a composite. From Table 2 in Appendix A, the stress exponents are greater than one, so the viscoelastic behavior of the materials tested is non-linear and the creep compliance $J(t)$ is not independent of stress. Resulting stress-strain-time relationships become extremely complex. If a non-linear viscoelastic "equation of state" could be developed, it might be too complicated for practical use, even with digital computer techniques.

In order to predict the behavior of composites studied in this investigation, a simplifying assumption was made. The strain response of a material was assumed to be independent of its stress-time history. Equation (7.2) then functions as an equation of state; if a stress and time are specified, then the strain is determined. From the physical restrictions placed on the composite tested in creep,

$$P_a + P_b = P \tag{7.9}$$

$$\epsilon_a = \epsilon_b = \epsilon \tag{7.10}$$

where P represents applied load and the subscripts a and b are used to identify quantities pertaining to component A or component B of the composite. Letting a and b denote the relative fraction of component A and component B respectively,

$$a + b = 1 \tag{7.11}$$

$$a\sigma_a + b\sigma_b = \sigma \tag{7.12}$$

Unsubscripted symbols above pertain to the composite structure as a whole. From equation (7.2),

$$\epsilon_b = K_b\sigma_b^{N_b} + B_b\sigma_b^{R_b}\cdot t + P_b\sigma_b^{M_b}\left(1 - e^{-C_b t^{Q_b}}\right) \tag{7.13}$$

$$\epsilon_a = K_a\sigma_a^{N_a} + B_a\sigma_a^{R_a}\cdot t + P_a\sigma_a^{M_a}\left(1 - e^{-C_a t^{Q_a}}\right) \tag{7.14}$$

Equation (7.12) may be solved for σ_a in terms of known composite stress σ, known volume fractions a and b, and the unknown stress σ_b. With this result substituted for σ_a in equation (7.14), equations (7.13) and (7.14) may be equated according to equation (7.10). For a specified time, the corresponding value of σ_b may be found. A series of values of σ_b and time can then be substituted in equation (7.13) to determine strain ϵ_b according to the equation of state assumption. By equation (7.10), strain ϵ_b is the same as the strain in the composite.

A digital computer program, program COMCUR, was used to predict composite behavior in this manner. Sample output from COMCUR consisting of point-by-point comparisons of predicted and observed strain is found in Appendix C, along with graphical displays of this output. A summary of results of this prediction method are tabulated in Table 4 of Appendix A. When the viscoelastic responses are similar, the load distribution in the components of the composite does not undergo large changes during the test and predictions are excellent. Such is the case for the polyethylene/polypropylene (PE/PP) composites. It should be noted that if the stress in the components were constant, the method would be theoretically correct, since equations (7.13) and (7.14) are valid for a constant stress test. Large errors in predictions occur with component materials of nylon and polycarbonate. As indicated by the magnitude of the creep parameters given in Table 2, Appendix A, the viscoelastic response of these materials is quite different. Graphical output from COMCUR shows that the load distribution for the NY/PC composites changes rapidly during the first few minutes of the test.

The empirical creep equation (Equation 7.2) was used to predict stress-strain behavior and stress relaxation behavior for single materials. This analysis was done with a digital computer using the equation of state assumption. An explanation of program RELAX, used to predict stress relaxation behavior is given in Appendix D. Output from program SIGEPS, an estimate of stress-strain curves, is given in Appendix E. In both cases, computed responses were considerably different from data obtained by experiment.

8. Conclusions.

The results of computer programs RELAX and SIGEPS demonstrate the complex nature of viscoelastic behavior. In tensile tests and relaxation experiments the stress changes with time and the constant stress equation gives poor predictions of responses. While the equation gives excellent estimates of creep behavior, the stress-time history cannot be ignored when predicting viscoelastic behavior in other modes of testing.

The combined mathematical and experimental approach to understanding composite behavior is a powerful tool. An analysis of simple composite structures provides guidelines for predicting behavior of more complicated systems where factors such as component distribution and interfacial shear stresses become important. The non-linear behavior of many viscoelastic materials, even at moderate stresses, leads to involved mathematical relationships. A digital computer is necessary for analysis of these complex systems.

Some opportunities for further investigation are:

(1) Use of X-ray diffraction techniques to follow structural changes during testing in order to determine the relation of internal structure change to observed non-linear viscoelastic effects.

(2) Fabrication and testing of more realistic composite materials to determine if the empirical equation developed here is general enough to characterize the behavior of these materials in creep.

(3) Use of other modes of testing, such as stress relaxation and dynamic mechanical tests to assist in development of non-linear viscoelastic theory.

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APPENDIX A

TABULATION OF RESULTS

Table 1

Properties of Tested Polymers

Material	*Tensile Strength psi	*Tensile Modulus psi x 10^{-3}	*Percent Elong.	Melt. Point °C	Glass Transition T_g, °C.
POLYETHYLENE	4000	220	380	137	-120
POLYPROPYLENE	4400	190	210	176	-10
POLYCARBONATE	8800	300	60	267	150
NYLON	5900	120	185	227	40

*At strain rate = 0.012 in/in/min

Table 2

Creep Parameters at Various Stress Levels - Single Specimens

POLYETHYLENE

Run No.	Stress psi	A_1 in/in$\times10^2$	A_2 in/in/hr$\times10^5$	A_3 in/in$\times10^2$	C	Q	Standard Deviation in Strain
2AE03	250	.0945	.542	.191	.7735	.3877	.00004
2AE05	500	.2017	1.160	.571	.9407	.3544	.00013
2AE07	750	.3144	.715	1.167	.6021	.3873	.00012
2AE08	750	.3144	.716	1.504	.9011	.2927	.00018
2AE10	1000	.4308	3.701	2.153	.8354	.4207	.00026
2AE13	1250	.5500	4.831	3.233	.7543	.4315	.00021
2AE15	1500	.6715	12.920	5.143	.7306	.4415	.00016
2AE18	1750	.7949	11.420	7.115	.6999	.4380	.00024

$$\epsilon = A_1 + A_2 t + A_3\left(1 - e^{-ct^{Q}}\right)$$

Table 2 - continued

POLYCARBONATE

Run No.	Stress psi	A_1 in/inX10^2	A_2 in/in/hrX10^5	A_3 in/inX10^2	C	Q	Standard Deviation in Strain
PC20A	2000	.5929	.2303	.00504	1.3903	.2119	.00006
PC25A	2500	.7665	.2711	.09558	.9049	.3181	.00007
PC30A	3000	.9455	.3718	.11942	.8514	.3913	.00010
PC35A	3500	1.1291	.4242	.18487	.6073	.2969	.00017
PC40A	4000	1.3172	.5644	.24303	.4874	.4949	.00005
PC45A	4500	1.5083	.6905	.29841	.6692	.4216	.00004
PC50A	5000	1.7020	.9782	.33040	.8591	.3300	.00009
NYLON 6/10							
NY10A	1000	.5660	1.822	1.373	.7068	.3443	.00031
NY15A	1500	.8610	1.768	2.799	.7425	.3141	.00012
NY20A	2000	1.1592	2.936	3.475	.8851	.3793	.00029
NY25A	2500	1.4608	3.220	5.906	1.0512	.2892	.00049
NY30A	3000	1.7640	3.914	8.505	1.1978	.2612	.00062
NY35A	3500	2.0691	5.136	10.820	1.1725	.2299	.00043

Table 2 - continued

POLYPROPYLENE

Run No.	Stress psi	A_1 in/in$X10^2$	A_2 in/in/hr$X10^5$	A_3 in/in$X10^2$	C	Q	Standard Deviation in Strain
2AP03	250	.0665	.2091	.1466	.5407	.2582	.00042
2AP05	500	.1580	.2221	.2862	.6571	.3413	.00013
2AP08	750	.2621	1.178	.6008	1.3590	.2289	.00029
2AP10	1000	.3752	2.340	.8994	.8017	.3764	.00028
2AP13	1250	.4957	2.963	.9487	.7313	.3987	.00021
2AP15	1500	.6224	4.232	1.5310	.7327	.4190	.00022
2AP20	2000	.8911	7.798	3.0102	.8500	.3563	.00032
2AP25	2500	1.1770	21.450	6.0931	.6956	.3395	.00019

Table 3

Correlated Stress Exponents and Coefficients

Material	K	N	B	R
POLYETHYLENE	$.22419 \times 10^{-5}$	1.0946	$.23176 \times 10^{-9}$	1.7449
POLYPROPYLENE	$.67718 \times 10^{-6}$	1.2479	$.41844 \times 10^{-8}$	1.2118
POLYCARBONATE	$.94095 \times 10^{-6}$	1.1510	$.59393 \times 10^{-10}$	1.3776
NYLON	$.44554 \times 10^{-5}$	1.1684	$.35936 \times 10^{-8}$	1.1684

Material	P	M	C	Q
POLYETHYLENE	$.12320 \times 10^{-6}$	1.7220	.8277	.3874
POLYPROPYLENE	$.86185 \times 10^{-7}$	1.6800	.8122	.3230
POLYCARBONATE	$.39496 \times 10^{-10}$	2.1645	.7799	.3964
NYLON	$.16992 \times 10^{-6}$	1.6397	.9593	.3030

$$\epsilon = K\sigma^{N} + B\sigma^{R} \cdot t + P\sigma^{M}(1 - e^{-ct^{Q}})$$

Table 4

Predicted Behavior of Composites in Creep

Run No.	Comp. A	Comp. B	%A	%B	Stress psi	Stress Distribution in Components at Various Elapsed Times, $\sigma A/\sigma B$			Error in Predicted Strain at t=100 hrs., percent	Predicted Strain Was:
						t=0	t=2 mins.	t=100 hrs		
1AA05	PE	PP	49.3	50.7	500	452/547	442/556	418/580	6.7	High
1AA11	PE	PP	48.9	51.1	1000	943/1054	899/1096	816/1176	0.7	High
1AA18	PE	PP	48.4	51.6	1750	1698/1795	1582/1907	1398/2080	5.3	Low
1AB15	PE	PP	66.1	33.9	1500	1465/1567	1402/1692	1284/1921	2.8	Low
1AB08	PE	PP	65.9	34.1	750	786/678	802/650	833/589	4.2	Low
XD25A	NY	PC	79.7	20.3	2500	2022/4380	1730/5525	1396/6834	20.5	Low
XD30A	NY	PC	82.5	17.5	3000	2490/5402	2145/7032	1770/8798	19.5	Low
XC30A	NY	PC	63.4	36.6	3000	2090/4578	1668/5307	1239/6051	5.6	Low
RA15A	PP	PC	87.1	12.9	1500	1404/2145	1313/2765	1103/4181	8.4	High
RA25A	PP	PC	86.6	13.4	2500	2325/3630	2122/4945	1779/7157	5.7	High
QA15A	PE	PC	86.2	13.8	1500	1378/2268	1227/3207	914/5161	2.9	High

APPENDIX B

COMPUTER PROGRAM FOR DETERMINING CREEP PARAMETERS

A Fortran IV computer program, compatible with the IBM 360 system, was used to determine the creep parameters A_2, A_3, C, and Q in the equation:

$$\epsilon = A_1 + A_2 \cdot t + A_3(1 - e^{-ct^Q})$$

The program, called FITDATA, is entered with the predetermined parameter A_1 and the deformation-time data obtained from the creep tests. Computer output consists of the calculated parameters, tabulated values of observed and computed strain, and intermediate data used in the parametric determination. An off-line plotting routine, called SUBROUTINE DRAW, is used to display the output (experimental data and fitted curve) in graphical form. This subroutine was also used to plot the data for determining parameters C and Q in the equation above. Least squares fitting was done by the library subroutine, SUBROUTINE LSQPOL available at the NPGS Computer Facility. Program FITDATA, along with typical tabulated and graphical output, is given on the following pages. The program, discussed briefly in Section 7, is annotated with comment cards.

COMPUTER PROGRAM "FITDATA"

```
FORTRAN IV G LEVEL 1, MOD 1          MAIN           DATE = 68139        08/32/39              PAGE 0001
      IMPLICIT REAL*8 (A-H), REAL *8 (O-Z)
      REAL*8 ITITLE(12), KTITLE(12), LABA/8H        /
      DIMENSION EP(100),T(100), TZ(100),EPZ(100),Y(100),             DELY(1
     100), B(5),SB(5),TQ(5),ST(5),CQ(5),SC(5),AQ(30,30),ED(100),TLN(100)
     2,EOZ(100),TLNZ(100),TGG(200),YG1(100),ERR(100),ERRF(100),FED(100),
     3EPO(100),TO(100),A(30,30),TT(100),X(100),F2(100),C(5),EDZZ(100)
      REAL LABEL(50),LABB,TG(200),YG(200),YGM(200),TS(100),E(100),TA(100
     1),EM(100),DIFFT,DIFEP,DIST,TSKAL,EPSKAL,R,ZILCH/4H    /,
     2PMKP/4H   P/,PMARK(100),TY(100),TLNY(100),TYS(100),EOS(100),TLO,
     3FLAG,CZ,ZSCAL,YSCAL,TU(100),YU(100),EDLO, W(100)
      REAL AA1,AA2,AA3,CCC,QQQ
C
C               FOR EACH COMPUTER RUN THE FOLLOWING INFORMATION IS READ IN
C                  NOAT = NUMBER OF SETS OF DATA
C                    IX = NUMBER OF SETS OF OATA TO BE ORAWN ON EACH GRAPH.
C                    JC = COUNTING SYMBOL TO CHANGE THE POINT PLOTTING
C                         SYMBOL FOR EACH DATA SET.
C                  LABEL = RUN NUMBERS USED TO LABEL EACH DRAWN CREEP
C                           CURVE.
C                  ITITLE = TITLE OF GRAPH OF CREEP CURVES
C                  KTITLE TITLE EOR GRAPH SHOWING C AND Q DETERMINATION.
C
      READ 16, NDAT,IX
   16 FORMAT(2I5)
      READ 17,(LABEL(I), I = 1,NDAT)
   17 FORMAT(20A4)
      READ 18,(ITITLE(I),I = 1,12)
   18 FORMAT(6A8)
      NSKIP = 0
      READ 502, (KTITLE(I), I = 1,12)
  502 FORMAT(6A8)
  503 AIX = IX
      FNDAT = NOAT
      PAGE = FNDAT/AIX
      NPAGE = PAGE
      NDO = NPAGE*IX
      KC = 0
      DO 41  K = 1,NPAGE
C
C               A NEW GRAPH SHEET IS STARTED EACH TIME THE INDEX OF THIS
C               DO LOOP IS CHANGED.
C
      JC = 1
   40 DO 42 L = 1,IX
      JC = JC +1
      KC = KC + 1
      PRINT 10
   10 FORMAT(1H1)
C
C               THE FOLLOWING DATA IS READ IN:
C                  NPTS = NUMBER OF POINTS IN THE DATA SET.
C                  GL = TEST GUAGE LENGTH
C                  A1 = PREDETERMINED ZERO TIME STRAIN
C                  TMAX = MAXIMUM TIME IN HOURS TO WHICH THE FITTED CURVE
C                         IS TO BE DRAWN.
C                  EPSKAL = STRAIN SCALE ON CREEP CURVE GRAPH (IN/IN/INCH)
C                  TSKAL = TIME SCALE ON CREEP CURVE GRAPH (HOURS/INCH)
C                  CONST = CONSTANT USED TO CONVERT INPUT CREEP DATA
C                           TO STRAIN
C                  RUN = RUN DESIGNATOR USED TO IDENTIFY THE DATA
C                  EP = ARRAY OF CREEP DEFORMATION DATA: USUALLY READ IN
C                       AS DEFORMATION IN MILS
C                  TT = ARRAY OF TIMES CORRESPONDING TO THE ARRAY "EP"
C
      READ 23, NPTS,GL,A1,TMAX,EPSKAL,TSKAL,CONST,RUN
   23 FORMAT(I5,6F10.0,5X,A5)
      READ 25, (EP(I), I = 1,NPTS)
   25 FORMAT(14F5.1)
      READ 26, (TT(I), I = 1,NPTS)
   26 FORMAT(7F10.5)
C
C               CONVERT THE DEFORMATION DATA TO STRAIN IN INCHES/INCH.
C               MAKE THE INITIAL VALUE OF STRAIN EQUAL TO THE ZERO TIME
C               STRAIN AND THE INITIAL TIME EQUAL ZERO.  NUMBER OF DATA
C               POINTS IS INCREASED BY ONE, BECOMES "NPTS1".
C
      EPO(1) = A1
      TO(1) = 0.0
      DO 43  I = 1,NPTS
      EPO(I + 1) = EP(I)/(CONST*GL)
   43 TO(I+1) = TT(I)
      NPTS1 = NPTS + 1
      DO 308  I = 1,NPTS1
      EP(I) = EPO(I)
  308 TT(I) = TO(I)
C
C               THE NEXT SIXTEEN CARDS SELECT ELAPSED TIMES GREATER THAN
C               70% OF THE TOTAL TESTING TIME.  IF LESS THAN FOUR POINTS
C               ARE OBTAINED, THE NEXT LONGEST TIME IS SELECTED, AND THE
C               PROCESS IS CONTINUED UNTIL FOUR POINTS ARE OBTAINED.  IF
C               THE SHORTEST TIME IS LESS THAN 50% OF THE TOTAL ELAPSED
C               TIME, THE DATA IS DEEMED UNRELIABLE FOR DETERMINING
C               PARAMETER A2, AND IS DISCARDED.   THE SELECTED TIMES FOR
C               THE EXTRAPOLATION ARE STORED IN THE ARRAY "TZ" AND THE
C               CORRESPONDING STRAINS ARE STORED IN THE ARRAY "EPZ".
C
      TM = 0.5*TT(NPTS1)
      TF = 0.7*TT(NPTS1)
      KA = 0
  309 KA = KA + 1
      IF(TT(KA) - TF) 309,310,310
  310 KA = KA-1
      KB = NPTS1 - KA
      IF(KB - 4) 310,311,311
  311 IF(TT(KA+1).LT.TM)  GO TO 777
      GO TO 312
  777 PRINT 778, RUN
  778 FORMAT(///,10X,'DATA FOR RUN ',A5,' UNRELIABLE FOR STEADY-STATE CR
     1EEP EXTRAPOLATION',//)
      GO TO 48
  312 DO 313  I = 1,KB
      TZ(I) = TT(KA+I)
  313 EPZ(I) = EP(KA+I)
C
C               THE EXTRAPOLATION IS DONE USING LIBRARY SUBROUTINE
C               LSQPOL. THIS ROUTINE PERFORMS A FIRST DEGREE POLY-
C               NOMIAL FIT TO THE SELECTED POINTS.  COEFFICIENTS OF THE
C               FITTED STRAIGHT LINE ARE STORED IN THE ARRAY CALLED "B".
C               THE SUBROUTINE PRINTS OUT INPUT DATA AND THE RESULTS OF
C               THE FIT IN TABULAR FORM.  THE VALUES OF THE CREEP
C               PARAMETERS A2 AND A3 ARE THEN DETERMINED
C
      CALL LSQPOL(KB,1,0,0,0,SIGMA,TZ,EPZ,W,Y,DELY,B,SB,TQ,ST,CQ,SC,AQ)
      IF(SIGMA.GT.0.05)  GO TO 777
      A2 = B(2)
      A3 = B(1) - A1
```

```
C
C               COMPUTE THE DELAYED STRAIN FRACTION AS ARRAY "ED" AND
C               NATURAL LOGARITHM OF TIME AS THE ARRAY "TLN"
C
      DO 314  I = 1,NPTS1
  314 ED(I) = (EP(I) - (A1 + A2*TT(I)))/A3
      TLN(1) = 0.0
      DO 322  I = 2,NPTS1
  322 TLN(I) = DLOG(TT(I))
C
C               COMPUTE THE DATA REQUIRED TO DETERMINE CONSTANTS C AND Q.
C               DISCARD THE LAST TEN POINTS WHICH MAY GIVE ED VALUES
C               GREATER THAN ONE, DUE TO THE "FORCED FIT" OF PARAMETER A2.
C               ELIMINATE ANY VALUES IN THE INITIAL PART OF THE RUN WHERE
C               ED MAY BE LESS THAN ZERO BECAUSE OF AN OVERESTIMATE OF
C               PARAMETER A1.  ATORE THE RESULTS OF THE CALCULATION IN THE
C               ARRAYS "EDZZ" AND "TLNZ".
C
      LQ = 10
      IP = 0
  339 IP = IP + 1
      IF ((IP+LQ) - NPTS1) 340,340,341
  340 EDZ(IP) = ED(IP+LQ)
      TLNZ(IP) = TLN(IP + LQ)
      IF(1.-EDZ(IP)) 341,341,339
  341 KPTS = IP - 1
      DO 325  I = 1,KPTS
      EDZZ(I) = -DLOG(1. - EDZ(I))
  325 EDZZ(I) = DLOG(EDZZ(I))
C
C               USE SUBROUTINE LSQPOL TO PERFORM A LINEAR FIT TO THE DATA
C               "EDZZ" AND "TLNZ".  PRINT OUT THE DATA AND THE RESULTING
C               FIT.
C
      CALL LSQPOL(KPTS,1,0,0,0,SIGMA,TLNZ,EDZZ,W,Y,DELY,B,SB,TD,ST,CD,
     1SC,AD)
      DO 205  I = 1,KPTS
      FED(I) = B(1) + B(2)*TLNZ(I)
  205 ERRF(I) = ((EDZZ(I) - FED(I))*100.)/EDZZ(I)
      PRINT 71, RUN
   71 FORMAT(///,10X,'THE DATA USED TO FIND "C" AND "D" FOR RUN ',A5,
     1' IS LISTED BELOW:',///,6X,'DATA PT.   LN TIME     LN(-LN(1.-ED))
     2FITTED VALUES    PCT.ERROR',//)
      DO 210  I = 1,KPTS
      II = I + LQ
  210 PRINT 72, II,TLNZ(I),EDZZ(I),FED(I),ERRF(I)
   72 FORMAT(I12,F12.4,F13.4,F16.4,F13.2)
      IF(SIGMA.GT.0.1)  GO TO 891
      GO TO 893
  891 PRINT 892, SIGMA
  892 FORMAT(///,10X,'SIGMA = ',F5.3,'  UNRELIABLE FIT FOR FINDING C AND
     1 D',//)
C
C               FROM THE COEFFICIENTS OF THE FITTED POLYNOMIAL, COMPUTE
C               THE CREEP PARAMETERS C AND D.
C
  893 Q = B(2)
      CC = DEXP(B(1))
C
C               THE NEXT NINE STATEMENTS ASSIGN 200 VALUES TO THE ARRAY OF
C               TIMES "TGG" IN EQUAL INCREMENTS FROM ZERO TO TMAX.
C               VALUES OF FITTED STRAIN ARE COMPUTED IN FUNCTION SUBROGRAM
C               "EDN" USING THE DETERMINED CREEP PARAMETERS, AND STORED IN
C               THE ARRAY "YG".
C
      R = TMAX/200.
      TG(1) = 0.0
      TGG(1) = 0.0
      DO 44  I = 2,200
      TG(I) = TG(I-1) + R
   44 TGG(I) = TG(I)
      DO 45  I = 1,200
      YG(I) = EDN(A1,A2,A3,CC,D,TGG(I))
   45 YGM(I) = -YG(I)
C
C               THE NEXT EIGHTEEN STATEMENTS REDUCE THE DATA POINTS SO
C               THAT NO TWO DATA POINTS ARE CLOSER THAN 0.2 INCHES APART
C               ON THE GRAPH SHEET
C
      E(1) = EP(1)
      TA(1) = TT(1)
      JA= 1
      IZ = 1
      KR = 2
   60 JA = JA + 1
      IF(JA.GT.NPTS) GO TO 62
      TSCAL = TSKAL
      ESCAL = EPSKAL
      DIFFT = (TT(JA) - TT(IZ))/TSCAL
      DIFEP = (EP(JA) - EP(IZ))/ESCAL
      DIST = SQRT(DIFFT**2 + DIFEP**2)
      IF(DIST - 0.2) 60,61,61
   61 TA(KR) = TT(JA)
      E(KR) = EP(JA)
      KR = KR + 1
      IZ = JA
      GO TO 60
C
C               THE FOLLOWING STATEMENTS (DOWN TO STATEMENT 50) FURTHER
C               REDUCE THE NUMBER OF DATA POINTS TO THIRTY OR LESS, AS
C               REQUIRED BY SUBROUTINE DRAW.
C
   62 KR = KR -1
      IF(KR.LE.30) GO TO 50
      IF(KR.LE.59)  GO TO 63
      KD = KR/20
      KPLOT = KR/KD
      DO 47  I = 1,KPLOT
      E(I) = E(I*KD)
   47 TA(I) = TA(I*KD)
      KR = KPLOT
      GO TO 50
   63 KD = KR - 30
      KDP1 = KD + 1
      DO 64  I = 1,KD
      TA(I) = TA(I*2)
   64 E(I) = E(I*2)
      DO 65  I = KDP1,30
      TA(I) = TA(I+KD)
   65 E(I) = E(I+KD)
      KR = 30
C
C               THE INTEGER "NSKIP" DETERMINES WHICH GRAPH PLOTS WILL BE
C               EXECUTED
C                  NSKIP = 0:  DRAW ONLY THE CREEP CURVES
C                  NSKIP = 1:  DRAW ONLY THE CURVES FOR DETERMINING C AND D
C
   50 IF(NSKIP.NE.1)  GO TO 818
      DO 811  I = 1,KPTS
      TYS(I) = TLNZ(I)
  811 EDS(I) = EDZZ(I)
C
C               THE FOLLOWING STATEMENTS COMPUTE GRAPH SCALES FOR PLOTTING
C               THE C AND D DETERMINATION.  THE NUMBER OF DATA POINTS
C               ARE REDUCED TO THIRTY OR LESS.
C
      TLO = ABS(TYS(1))
      ITLO = TLO*10.
      TLC = ITLO + 1
      TLC = TLC/10.
      IF(TYS(1).GT.0.0)  TLO = -TLO
      EDLO = ABS(EDS(1))
      IEDLO = EDLO*10.
      EDLO = IEDLO + 1
      EOLC = EOLD/10.
      IF(EOS(1).GT.0.0)  EDLD = -EOLO
      FLAG = 0.0D0001
      QZ = A8S(ITYS(KPTS) - TYS(1))/8.)
      QZ = QZ*1000000.
  812 QZ = QZ/10.
      FLAG = FLAG*10.
      IF(QZ - 10.) 813,812,812
  813 IQZ = QZ + 1.
      ZSCAL = IQZ
      ZSCAL = ZSCAL*FLAG
      FLAG = 0.000001
      QZ = A8SI(EDSIKPTS) - EDS(1))/8.)
      OZ = QZ*1000000.
  814 OZ = QZ/10.
      FLAG = FLAG*10.
      IF(QZ - 10.) 822,814,814
  822 IQZ = QZ + 1.
      YSCAL = IQZ
      YSCAL = YSCAL*FLAG
      OD 823  I = 1,KPTS
      TYS(I) = TYS(I) + TLO
  823 EDS(I) = EDS(1) + EOLO
      MR = KPTS
      (F(MR.LE.30)  GD TD 80
      IF(MR.LE.59)  GO TD 76
      MQ = MR/20
      MPLOT = MR/MO
      DD 75  ( = 1,MPLOT
      EDS(1) = EOS(I*MO)
   75 TYS(1) = TYS(1*MO)
      MR = MPLOT
      GD TO 80
   76 MD = MR - 30
      MDP1 = MD + 1
      DD 78  ( = 1,MD
      EDS(1) = EOS(I*2)
   78 TYS(I) = TYS(I*2)
      DD 79  I = MOP1,3D
      EDS(I) = EDS(I+MD)
   79 TYS(1) = TYS(I+MD)
      MR = 30
   80 CDNTINUE
      LA88 = LA8EL(KC)
C
C              MAKE A PLOT DF THE OATA USED TD OETERM(NE C ANO Q.
C
      CALL ORAW(MR,TYS,EDS,1,1,LA8A,KT(TLE,ZSCAL,YSCAL,1,1,2,2,9,9,1,LAS
     1TC)
      PRINT 815, LASTC
  815 FDRMAT(//,10X,'LASTC = ',(2)
      TLOW = TLO
      EOLOW = EDLD
      OD 824  I = 1,KPTS
      TU(I) = TLNZ(I) + TLOW
  824 YU(I) = FEO(I) + EDLDW
C
C              SUPFRIMPDSE DN THE OATA THE FITTEO CUPVE FOUND FRDM
C              SU8RDUTINE LSOPOL.
C
      CALL ORAW(KPTS,TU,YU,3,0,LA88,KT(TLE,ZSCAL,YSCAL,1,1,2,2,9,9,1,LAS
     1TD)
      PR(NT 816, LASTO
  816 FDRMAT(//,10X,'LASTO = ', I2)
      PRINT 817,TLD,EOLO
  817 FORMAT(///,10X,'FDR THF PLDT USED TO DETERM(NE"C" AND "O", THE SCA
     1LES MUST 8E CDRRECTED AS FDLLDWS:',//,30X,'SUBTRACT',F4.2,'FROM X-
     2SCALE VALUES',/,30X,'SU8TRACT',F4.2,'FRDM Y-SCALE VALUFS',//)
C
C              IF THE PREV(DUS GRAPHS HAVE 8EFN FXECUTED, OD NOT DRAW
C              CREEP CURVES 8UT GD TD STATEMENT 819 AND PRINT OUT THE
C              RESULTS DF THE EMP(RICAL CREEP FIT.
C
      GO TD 819
  818 CDNTINUE
C
C              THE NEXT THREE STATEMENTS PRDVIDE THE PRDPFR VALUE OF THE
C              CALLING ARGUMENTS "JC" AND "MODCUR" SD THE CPEEP CURVES
C              MAY 8E PLDTTED.
C
      (F(JC.GT.5)  JC = JC - 5
      IF(L.EQ.1) MDDCUR = 1
      IF(L.GT.1) MDDCUR = 2
      OD 66  ( = 1,KR
   66 EM(1) = - E(I)
      CALL DRAW(KR,EM,TA,MDDCUR,JC,LA8A,(TITLE,EPSKAL,TSKAL,C,8,2,2,8,15
     1,1,LASTA)
      PRINT 29, LASTA
   29 FDRMAT(//,10X,'LASTA = ', I2)
      (F(L.EQ.IX) MODCUR = 3
      (F(L.LT.IX) MODCUR = 2
      LA88 = LA8EL(KC)
      CALL DRAW(200,YGM,TG,MDOCUR,0,LA8B,(TITLE,EPSKAL,TSKAL,0,8,2,2,8,1
     15,1,LAST8)
      PRINT 30, LASTB
   30 FORMAT(//,10X,'LASTB = ', (2,////)
  819 CDNTINUE
C
C              F(TTFD PARAMETERS ARE PR(NTFD AS OUTPUT.
C
      PR(NT 27
   27 FORMAT(5X,'THF PARAMETERS DF THE FITTED CURVF ARE:',//)
      PRINT 33
   33 FDRMAT(9X,'A1',12X,'A2',12X,'A3',12X,'C',13X,'O',/)
      PRINT 28, A1,A2,A3,CC,Q
   28 FORMAT(1H ,3D14.4,2F14.4,///)
      AA1 = A1
      AA2 = A2
      AA3 = A3
      CCC = CC
      QQQ = Q
C
C              F(TTED PARAMETERS ARE PUNCHFD DN CAROS FOR USE IN
C              SUBSEQUFNT PRDGRAMS.
C
      PUNCH 992, AA1,AA2,AA3,CCC,QQQ
  992 FORMAT(3E15.5,2F10.5)
      DD 560  ( = 1,NPTS1
  560 TS(I) = TT(I)
C
C              THE STATEMENTS ODWN TD STATEMENT 564 ARRANGE FOR A
C              SYMBOL "P" TD 8E PRINTED (N THF TABULATFD DATA IF AN
C              EXPER(MENTAL DATA PDINT WAS PLOTTED DN THE CREFP CURVFS.
C
      OD 564  ( = 1,NPTS1
      J = 1
  561 VAL = A8S(TS(I) - TA(J))
      IF(VAL - 0.000C1) 562,562,563
  562 PMARK(() = PMKP
      GO TO 564
  563 PMARK(() = Z(LCH
      J = J + 1
      IF(J.GT.KR)  GO TD 564
      GO TO 561
  564 CONT(NUE
      DD 51  I = 1,NPTS1
      YG1(I) = EQN(A1,A2,A3,CC,Q,TT(I))
   51 ERR(I) = ((YG1(I) - EP(I))*10C.)/EP(I)
```

```
      C
      C             PRINT OUT THE OBSERVED STRAIN, FITTED STRAIN, AND
      C             PERCENT ERROR IN A POINT-BY-POINT TABULATION.
      C
                PRINT 31, RUN
             31 FORMAT(10X,'THE DATA FITTED AND PLOTTED FOR RUN ',A5,' IS LISTED B
               1ELOW',///,6X,'POINT NO.     TIME,HOURS      OBSERVED STRAIN      CALC
               2 STRAIN      PCT ERROR      "P" IF PLOTTED',///)
                PRINT 32, (I,TT(I),EP(I),YG1(I),ERR(I),PMARK(I),   I = 1,NPTS1)
             32 FORMAT(1H0,I10,F18.5,7X,F10.5,9X,F10.5,5X,F8.3,A12)
             48 CONTINUE
             42 CONTINUE
             41 CONTINUE
      C
      C             THE NEXT FOUR STATEMENTS CAUSE ANY "LEFTOVER" DATA SETS TO
      C             BE FITTED AND PLOTTED BEFORE ENDING THE PROGRAM.
      C             AT THIS POINT THERE ARE BETWEEN ZERO AND (IX - 1) DATA
      C             SETS NOT YET ANALYZED.
      C
                IF(NDAT - NDO) 53,53,52
             52 IX = NDAT - NDO
                NDO = NDAT
                GO TO 40
             53 CONTINUE
                END
```

```
FORTRAN IV G LEVEL 1, MOD 1          EQN              DATE = 68139          08/32/39                PAGE 0001

                 FUNCTION EQN(A1,A2,A3,CC,D,Z)
                 IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
                 IF(Z.EQ.0.D)  GO TO 704
                 PWR = CC*(Z**D)
                 IF(PWR - 50.) 702,701,701
             701 TERM = 1.0
                 GO TO 703
             702 TERM = 1. - DEXP(-PWR)
             703 EQN = A1 + A2*Z + A3*TERM
                 GO TO 708
             704 EQN = A1
             708 CONTINUE
                 RETURN
                 END
```

OUTPUT FROM PROGRAM "FITDATA"

A_2 EXTRAPOLATION, FROM SUBROUTINE LSQPOL

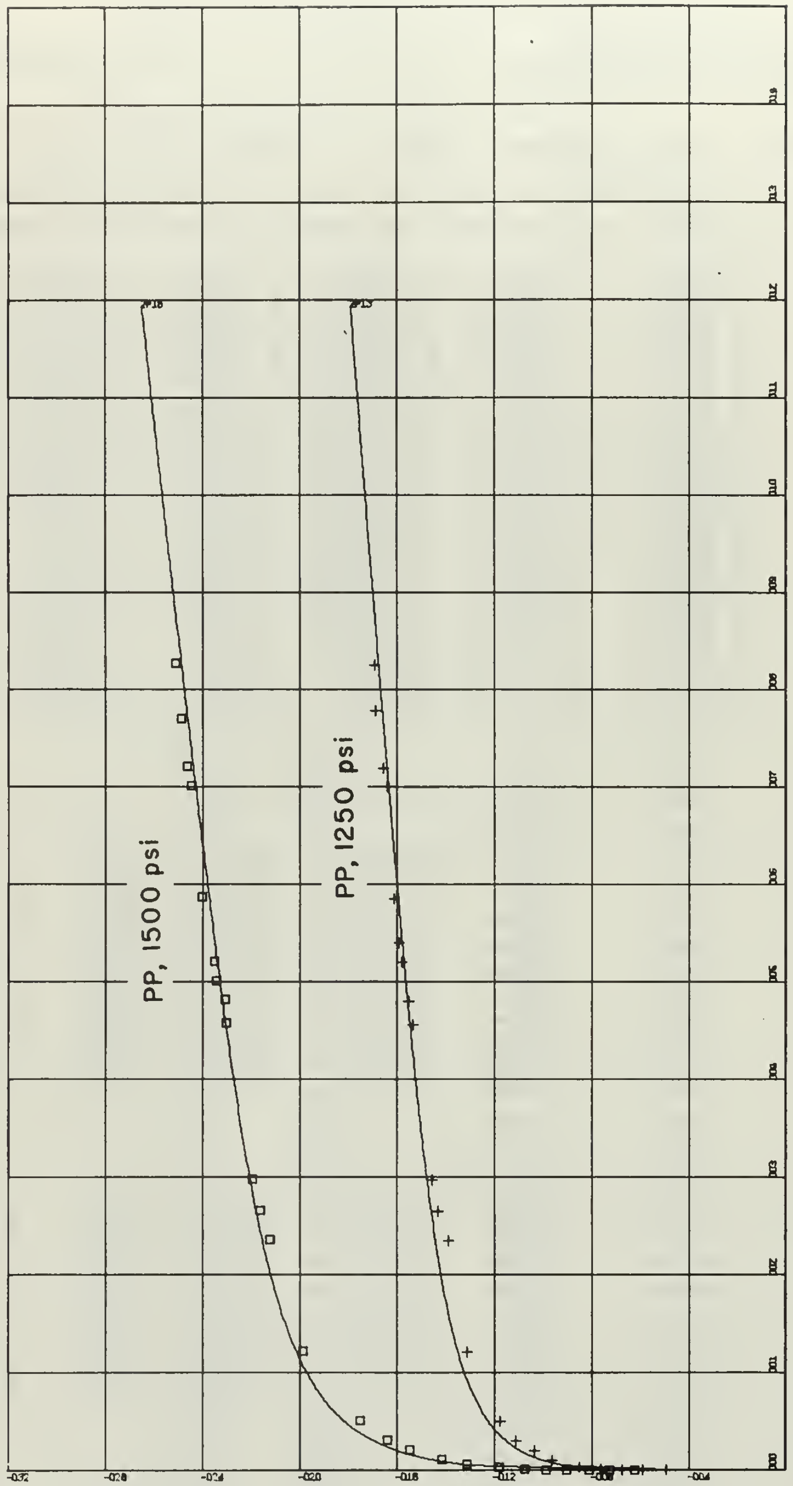
```
COEFFICIENTS OF Y=B1+B2*X+ETC AND ERRORS

B( 1)=   1.4443827D-02 ERRB= 4.065E-04 B( 2)=   2.9625553D-05 ERRB= 5.166E-06 B(

SIGMA=   1.0256402E-04 F LEVEL=    0.0                SUM SQ DEV=    7.5138438E-09

COEFFICIENTS OF Y=T1*P1+T2*P2+ETC AND ERRORS

T( 1)=   1.6764286D-02 ERRT= 3.877E-05 T( 2)=   2.9625553D-05 ERRT= 5.166E-06 T(

  I           X(I)               F(I)               Y(I)              DELY(I)              W(I)

    1    7.0000000D 01    1.6400000D-02    1.6517616D-02    1.1761616E-04    1.0000000E 00
    2    7.1966700D 01    1.6550000D-02    1.6575881D-02    2.5880727E-05    1.0000000E 00
    3    7.3833300D 01    1.6600000D-02    1.6631180D-02    3.1179792E-05    1.0000000E 00
    4    7.7867000D 01    1.6850000D-02    1.6750680D-02   -9.9319601E-05    1.0000000E 00
    5    7.8016700D 01    1.6900000D-02    1.6755115D-02   -1.4488466E-04    1.0000000E 00
    6    8.2600000D 01    1.6900000D-02    1.6890898D-02   -9.1018655E-06    1.0000000E 00
    7    9.4000000D 01    1.7150000D-02    1.7228629D-02    7.8629440E-05    1.0000000E 00
```

DETERMINATION OF PARAMETERS "C" AND "Q"

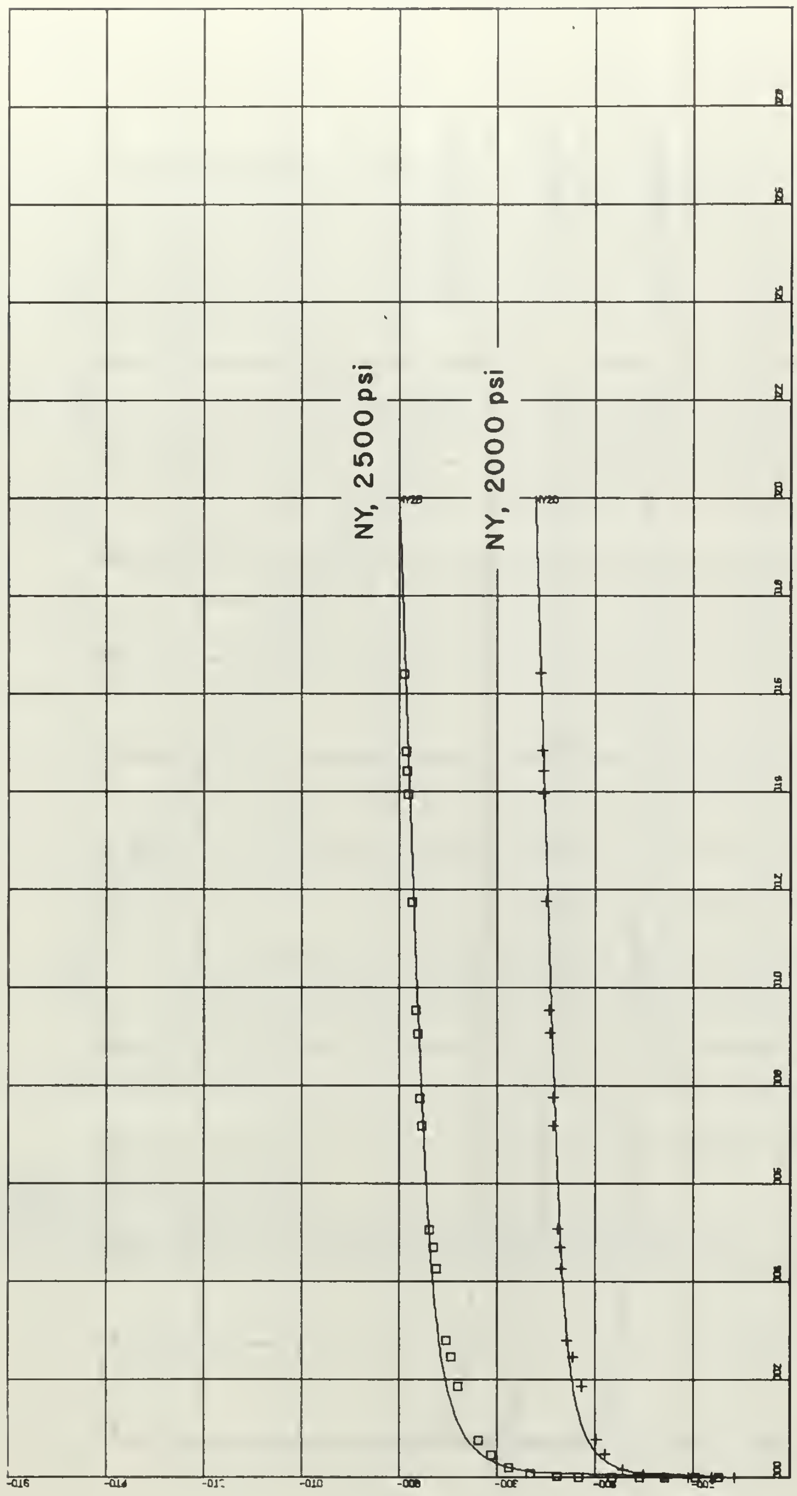
```
COEFFICIENTS DE Y=81+82*X+ETC AND ERRORS
B( 1)= -3.1293105D-01 ERRB= 2.330E-02 8( 2)=  3.9867907D-01 ERRB= 8.252F-03 B(
SIGMA=   1.4200133E-01 F LEVEL=   0.0             SUM SQ DEV=   1.9103099E-02
CDEEFICIENTS OE Y=T1*P1+T2*P2+ETC AND ERRORS
T( 1)= -1.42793440-01 ERRT= 2.304E-02 T( 2)=  3.9867907D-01 ERRT= 8.252E-03 T(
  I           X(I)            E(I)            Y(I)            OELY(I)          W(I)

   1  -4.09414460 00  -1.80231440 00  -1.94518080 00  -1.4286631E-01   1.00C0000E 00
   2  -3.68887950 00  -1.66040580 00  -1.78361010 00  -1.2320423E-01   1.0000000E 00
   3  -3.40129740 00  -1.56381250 00  -1.66895710 00  -1.0514462E-01   1.0000000F 00
   4  -3.17821380 0C  -1.47407730 00  -1.58001840 00  -1.0594100E-01   1.00C0000E 0C
   5  -2.99573230 00  -1.44559500 00  -1.50726680 00  -6.1671790E-02   1.00C0000E 0C
   6  -2.84181020 00  -1.39028630 00  -1.44590130 00  -5.5614956E-02   1.00000C0E 0C
   7  -2.70800020 00  -1.36353890 00  -1.39255400 00  -2.9015187E-02   1.0000000E 00
   8  -2.48494670 00  -1.28620760 00  -1.30362730 00  -1.7419655E-02   1.0000000E 00
   9  -2.30258510 00  -1.18930850 00  -1.23092350 00  -4.1614994E-02   1.0000000E 00
  10  -2.14840580 00  -1.16625190 00  -1.16945550 00  -3.2035466E-03   1.0000000E 00
  11  -2.01492800 00  -1.12098360 00  -1.11624070 00   4.7428943E-03   1.0000000E 00
  12  -1.89712000 00  -1.05545600 00  -1.06927310 00  -1.3817094E-02   1.0000000E 00
  13  -1.69646750 00  -9.92817350-01  -9.89277120-01   3.5402330E-C3   1.0000C00E 00
  14  -1.32174330 00  -8.55536100-01  -8.39882450-01   1.5653651E-02   1.00C0000E 00
  15  -1.04982210 00  -7.64026990-01  -7.31473150-01   3.2553840E-02   1.0000000F 00
  16  -8.36255720-01  -7.29183090-C1  -6.46328700-01   8.2854390E-02   1.0000000E 00
  17  -6.60350910-01  -6.60462630-01  -5.76199130-01   8.4263444E-02   1.0000000E 00
  18  -2.65698820-01  -5.14083030-01  -4.18859600-01   9.5223427E-02   1.0000000E 0C
  19   1.65325810-02  -4.06496960-C1  -3.06339850-01   1.0015708E-01   1.0000000E 00
  20   4.16536920-01  -2.89942540-01  -1.46866500-01   1.4307600E-01   1.00C0000F 00
  21   7.01462510-01  -1.91264420-01  -3.32726280-02   1.5799177E-01   1.0C00000E 0C
  22   1.10416350 00   1.62578570-02   1.27275830-01   1.1101794E-01   1.0000000E 00
  23   1.39046070 00   7.94881650-02   2.41416520-01   1.6192830F-01   1.0000C00E 0C
  24   1.61277230 0C   1.86659490-01   3.30047530-01   1.4338803E-01   1.C000C00E 0C
  25   1.79453890 00   2.96263960-01   4.02514060-01   1.0625005E-01   1.00C0000E 00
  26   2.49458470 00   5.40936280-01   6.81607640-01   1.4067131E-01   1.C0000C0E 00
  27   3.15771080 0C   6.90561770-C1   9.45982150-01   2.5542033E-01   1.00C00C0E 0C
  28   3.27777470 00   8.42902470-C1   9.93849120-01   1.5094662E-01   1.0000000E 00
  29   3.39170920 00   9.18610750-01   1.03927240 00   1.2066162E-01   1.0000000E 00
  30   3.82173280 00   1.11664430 00   1.21071380 00   9.4069481E-02   1.00C000CE 00
  31   3.87258960 00   1.17866040 00   1.23098940 00   5.2328981E-C2   1.00000C0E 00
  32   3.91235690 00   1.26375620 00   1.24684380 00  -1.6912397E-02   1.C0C000CE 00
  33   3.95156480 00   1.37069550 00   1.26247510 00  -1.0822034E-01   1.C0C0000E 00
  34   3.98929330 00   1.43201240 00   1.27751670 00  -1.5449566E-01   1.C0000C0E 0C
  35   4.07073470 00   1.56382370 00   1.30998570 C0  -2.5383794E-01   1.00C00C0E 00
  36   4.24849520 00   1.47937750 00   1.38085510 00  -9.8522425E-02   1.00C00C0F 0C
  37   4.27620350 00   1.77565370 C0   1.39190180 00  -3.8375187E-01   1.00000C0E 00
  38   4.30180980 00   1.74359600 C0   1.40211050 00  -3.4148550E-01   1.0C000C0E 0C

     THE OATA USEO TO E1NO "C" AND "O" EOR RUN 2AP13 IS LISTED 8ELOW:

  DATA PT.  LN TIME    LN(-LN(1.-ED))   EITTED VALUES   PCT.ERROR

     11       -4.0941      -1.8023            -1.9452         -7.93
     12       -3.6889      -1.6604            -1.7836         -7.42
     13       -3.4013      -1.5638            -1.6690         -6.72
     14       -3.1782      -1.4741            -1.5800         -7.19
     15       -2.9957      -1.4456            -1.5073         -4.27
     16       -2.8418      -1.3903            -1.4459         -4.00
     17       -2.7080      -1.3635            -1.3926         -2.13
     18       -2.4849      -1.2862            -1.3036         -1.35
     19       -2.3026      -1.1893            -1.2309         -3.50
     20       -2.1484      -1.1663            -1.1695         -0.27
     21       -2.0149      -1.1210            -1.1162          0.42
     22       -1.8971      -1.0555            -1.0693         -1.31
     23       -1.6965      -0.9928            -0.9893          0.36
     24       -1.3217      -0.8555            -0.8399          1.83
     25       -1.0498      -0.7640            -0.7315          4.26
     26       -0.8363      -0.7292            -0.6463         11.36
     27       -0.6604      -0.6605            -0.5762         12.76
     28       -0.2657      -0.5141            -0.4189         18.52
     29        0.0165      -0.4065            -0.3063         24.64
     30        0.4165      -0.2899            -0.1469         49.35
     31        0.7015      -0.1913            -0.0333         82.60




     32        1.1042       0.0163             0.1273       -682.86
     33        1.3905       0.0795             0.2414       -203.71
     34        1.6128       0.1867             0.3300        -76.82
     35        1.7945       0.2963             0.4025        -35.86
     36        2.4946       0.5409             0.6816        -26.01
     37        3.1577       0.6906             0.9460        -36.99
     38        3.2778       0.8429             0.9938        -17.91
     39        3.3917       0.9186             1.0393        -13.14
     40        3.8217       1.1166             1.2107         -8.42
     41        3.8726       1.1787             1.2310         -4.44
     42        3.9124       1.2638             1.2468          1.34
     43        3.9516       1.3707             1.2625          7.90
     44        3.9893       1.4320             1.2775         10.79
     45        4.0707       1.5638             1.3100         16.23
     46        4.2485       1.4794             1.3809          6.66
     47        4.2762       1.7757             1.3919         21.61
     48        4.3018       1.7436             1.4021         19.59
```

TABULATION OF CALCULATED PARAMETERS AND RESULTING CURVE FIT

```
  THE PARAMETERS OF THE FITTED CURVE ARE:

      A1             A2             A3              C              D
  0.4957D-02     0.2963D-04     0.9487D-02       0.7313         0.3987

        THE DATA FITTED AND PLOTTED FOR RUN 2AP13 IS LISTED BELOW

   POINT NO.     TIME,HOURS      OBSERVED STRAIN     CALC STRAIN     PCT ERROR      "P" IF PLOTTED


       1            0.0               0.00496           0.00496         0.0             P
       2            0.00056           0.00550           0.00530        -3.611
       3            0.00139           0.00560           0.00545        -2.718
       4            0.00278           0.00575           0.00560        -2.633
       5            0.00417           0.00590           0.00571        -3.277           P
       6            0.00556           0.00600           0.00579        -3.440
       7            0.00694           0.00610           0.00587        -3.818
       8            0.00833           0.00615           0.00593        -3.542
       9            0.01111           0.00625           0.00604        -3.299
      10            0.01388           0.00630           0.00614        -2.569
      11            0.01667           0.00640           0.00622        -2.790
      12            0.02500           0.00660           0.00643        -2.648
      13            0.03333           0.00675           0.00659        -2.407           P
      14            0.04166           0.00690           0.00672        -2.546
      15            0.05000           0.00695           0.00684        -1.530



      16            0.05832           0.00705           0.00695        -1.423
      17            0.06667           0.00710           0.00705        -0.760
      18            0.08333           0.00725           0.00722        -0.475
      19            0.10000           0.00745           0.00736        -1.173
      20            0.11667           0.00750           0.00749        -0.092
      21            0.13333           0.00760           0.00761         0.140           P
      22            0.15000           0.00775           0.00772        -0.414
      23            0.18333           0.00790           0.00791         0.109
      24            0.26667           0.00825           0.00829         0.502
      25            0.35000           0.00850           0.00859         1.071           P
      26            0.43333           0.00860           0.00884         2.779
      27            0.51667           0.00880           0.00905         2.855
      28            0.76667           0.00925           0.00955         3.271
      29            1.01667           0.00960           0.00993         3.440           P
      30            1.51670           0.01000           0.01049         4.880
      31            2.01670           0.01035           0.01090         5.290           P
      32            3.01670           0.01110           0.01149         3.479           P
      33            4.01670           0.01135           0.01191         4.906
      34            5.01670           0.01175           0.01223         4.102           P
      35            6.01670           0.01215           0.01250         2.847
      36           12.11670           0.01310           0.01349         2.970           P
      37           23.51670           0.01385           0.01442         4.103           P
      38           26.51670           0.01430           0.01459         2.048           P
      39           29.71670           0.01455           0.01476         1.462           P
      40           45.68330           0.01535           0.01547         0.758           P
      41           48.06670           0.01550           0.01556         0.380           P
      42           50.01670           0.01565           0.01563        -0.108
      43           52.01670           0.01580           0.01571        -0.582           P
      44           54.01670           0.01590           0.01578        -0.744           P
      45           58.60000           0.01610           0.01595        -0.952           P
      46           70.00000           0.01640           0.01634        -0.366           P
      47           71.96670           0.01655           0.01641        -0.870           P
      48           73.83330           0.01660           0.01647        -0.794
      49           77.86700           0.01685           0.01660        -1.476           P
      50           78.01670           0.01690           0.01661        -1.739
      51           82.60000           0.01690           0.01676        -0.855           P
      52           94.00000           0.01715           0.01712        -0.172
```

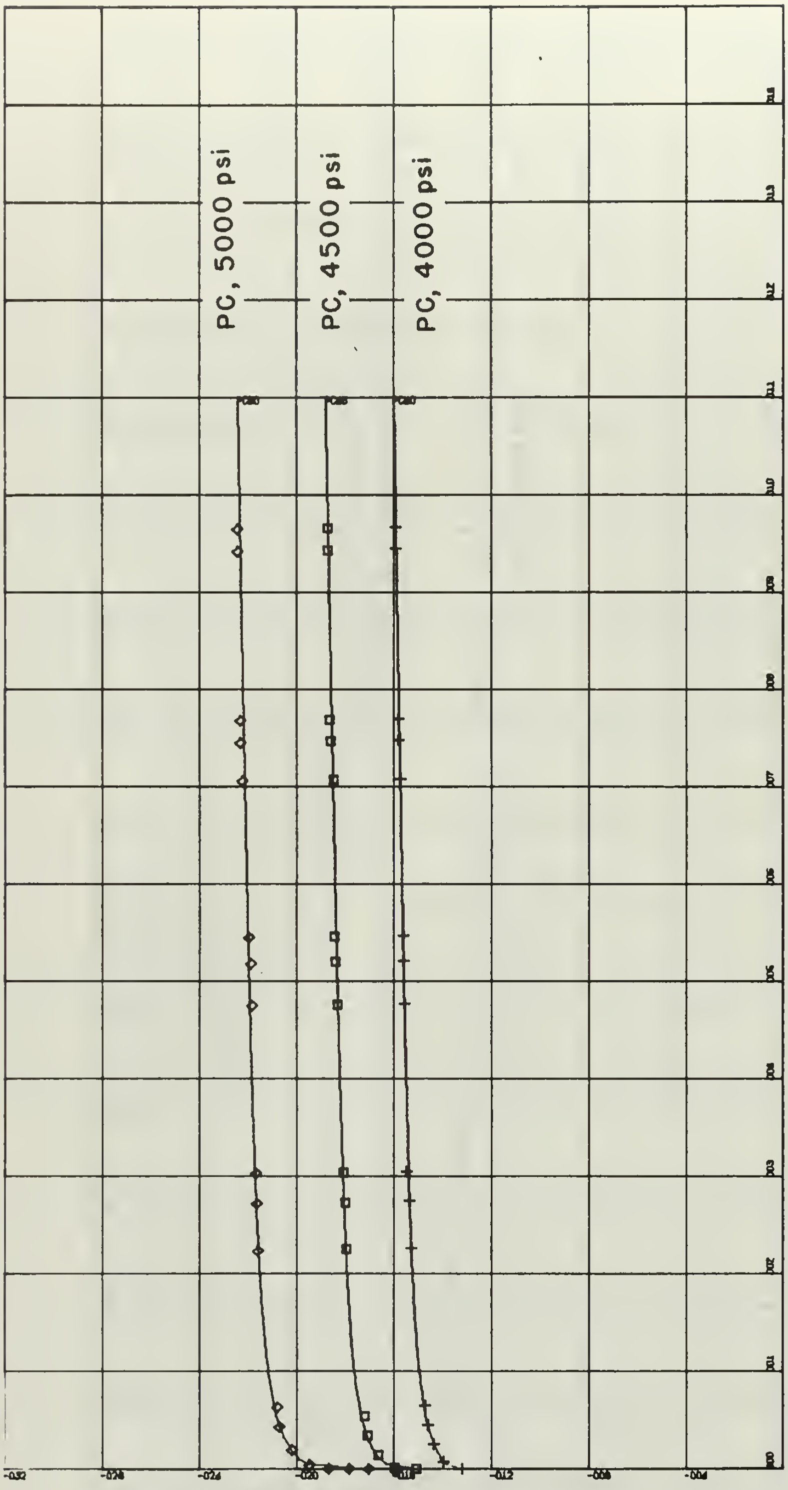


X-SCALE=4.00E-03 UNITS INCH.
Y-SCALE=1.00E+01 UNITS INCH.
HERING,F.S. BOX 21, CREEP OF POLYPROPYLENE
TIME(HOURS) VS. STRAIN(INCHES/INCH)



X-SCALE=2.00E-02 UNITS INCH.
Y-SCALE=2.00E+01 UNITS INCH.
HERING,F.S., BOX 21, CREEP OF NYLON
TIME(HOURS) VS. STRAIN(INCHES/INCH)

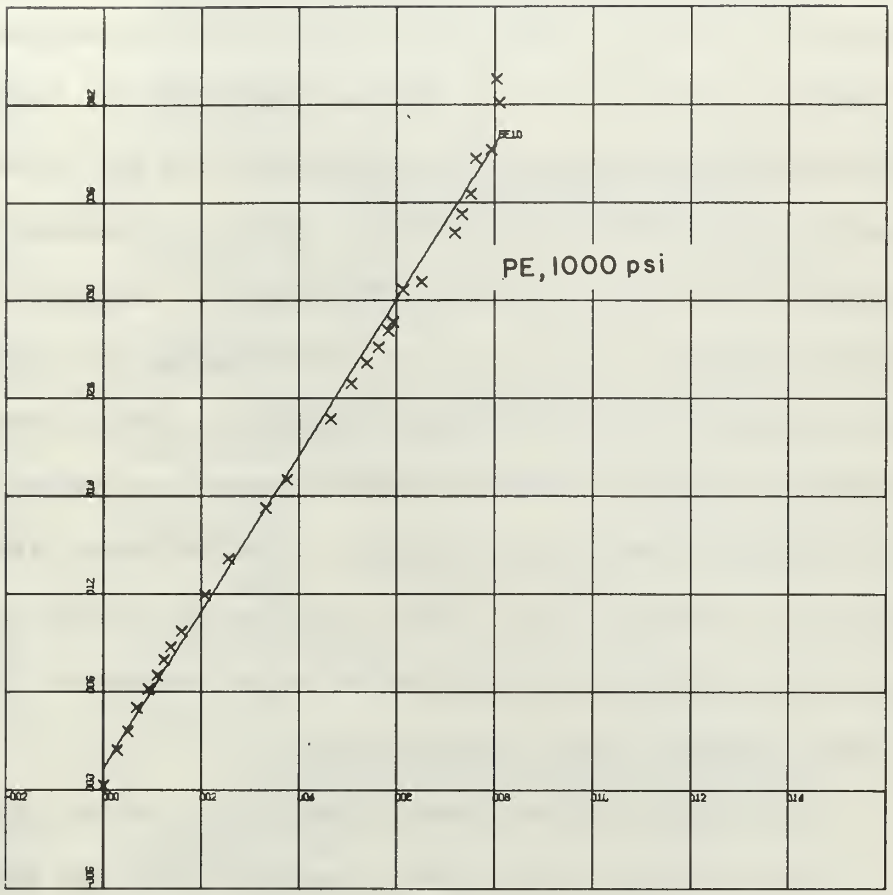
CALIFORNIA COMPUTER PRODUCTS, INC. ANAHEIM CALIFORNIA CHART NO



-SCALE=4.00E-03 UNITS INCH.
Y-SCALE=1.00E+01 UNITS INCH.
HERING, F.S. BOX 21, CREEP OF POLYCARBONATE
TIME, HOURS VS. STRAIN, INCHES/INCH

CALIFORNIA COMPUTER PRODUCTS, INC ANAHEIM, C

CALIFORNIA COMPUTER PRODUCTS, INC ANAHEIM, CALIFORNIA CHART



X-SCALE=2.00E+00 UNITS INCH.
Y-SCALE=6.00E-01 UNITS INCH.
HERING.F.S. BOX 21, CREEP OF POLYETHYLENE
RATE CONST.DETERMINATION LN(-LN(1-ED))VS.LN TIME

CALIFORNIA COMPUTER PRODUCTS INC. ANAHEIM CALIFORNIA CHART NO 00

APPENDIX C

COMPUTER PROGRAM FOR PREDICTING CREEP BEHAVIOR OF COMPOSITES

As discussed in Section 7, the prediction of composite behavior was made by assuming the analytic expression for creep of the components to be an equation of state. Input data to the Fortran IV computer program called COMCUR consists of the experimental creep data for the composite, the volume fractions a and b for each component, and the stress coefficients and exponents for each component. These were computed from stress correlations of the creep parameters for individual components obtained from program FITDATA. For a series of times ranging over the testing period, the stress in each component is computed. This is done by a Newton-Raphson iteration since the unknown cannot be expressed explicitly as a function of the other variables. With the stress distribution determined, the composite creep strain, equal to the strain in each component, is computed using the equation of state assumption. Subroutine DRAW is used to display the following output:

(1) Calculated creep curve of component A at the given stress

(2) Calculated creep curve of component B at the given stress

(3) Predicted creep curve of composite AB at the given stress for the specified volume fractions of components A and B

(4) Experimentally observed creep data for composite AB

The load distribution in the composite during the first 200 seconds of the test is also presented in graphical form. Tabulated output and graphs for a typical run are given on the following pages. The program COMCUR and the related function subprograms are annotated with comment cards.

COMPUTER PROGRAM "COMCUR"

```
      REAL*8 ITITLE(12),RUN,KTITLE(12)
      REAL KA,KB,NA,NB,MA,MB,K,M,N,LABEL(30),LABB
      DIMENSION TZ(200), STRA(200), STRB(200),ECOM(200), EA(200), EB(200
     1),EMEAS(200),TMEAS(200),FRACTA(200),FRACTB(200),ECALC(200),ERR(200
     2),EMEASM(200),EAM(200),EBM(200),ECOMM(200),TQ(25),SA(25),SB(25),FA
     3(25),FB(25),SIGG(200),TG(50),EG(50),TY(100),TS(100),DA(100),DB(100
     4)
C
C             READ IN TITLES FOR GRAPHICAL OUTPUT
C
      READ 5, (ITITLE(I),  I = 1,12)
      READ 5, (KTITLE(I),  I = 1,12)
    5 FORMAT(6A8)
      READ 6, (LABEL(I),  I = 1,3)
    6 FORMAT(3A4)
C
C             READ IN THE FOLLOWING DATA:
C                RUN = RUN NUMBER OF EXPERIMENTAL COMPOSITE TEST
C                NPTS = NUMBER OF CREEP DATA POINTS FOR THE COMPOSITE.
C                TMAX = MAX. TIME IN HOURS TO WHICH THE COMPUTED
C                       CREEP CURVES ARE TO BE DRAWN.
C                TSKAL = TIME SCALE(HOURS/INCH) OF CREEP GRAPHS
C                EPSKAL = STRAIN SCALE(IN/IN PER INCH) OF CREEP CURVFS
C
      READ 7, RUN,NPTS,TMAX,TSKAL,EPSKAL,GL
    7 FORMAT(A8,I12,4F10.0)
C
C             READ IN EXPERIMENTAL DATA
C                EMEAS = DEFORMATION IN MILS
C                TMEAS = TIME IN HOURS
C
      READ 8, (EMEAS(I),  I = 1,NPTS)
    8 FORMAT(14F5.1)
      READ 9,(TMEAS(I), I = 1,NPTS)
    9 FORMAT(7F10.5)
C
C             READ IN THE STRESS CORRELATED PARAMETERS OF THE EMPIRICAL
C             CREEP EQUATIONS FOR COMPONENTS A AND B
C
      READ 10, KA,BA,PA,QA,CA,NA,RA,MA
      READ 10,KB,BB,FB,QB,CB,NB,RB,MB
   10 FORMAT(3E15.5,5F6.4)
C
C             READ IN THE VOLUME FRACTIONS (A AND B) OF EACH COMPONENT
C             AND THE STRESS LEVEL OF THE EXPERIMENTAL TEST (STR)
C
      READ 11, A,B
   11 FORMAT(2F10.0)
      READ 12, STR
   12 FORMAT(F10.0)
C
C             GENERATE 200 TIME INTERVALS AT WHICH TO PREDICT THE CREFP
C             CURVE.
C
      TINC = TMAX/200.
      TZ(1) = TINC
      DO 31  I = 2,2C0
   31 TZ(I) = TZ(I-1) + TINC
      J = 1
C
C             ESTIMATE THE STRESS IN COMPONENT B AT THE START OF THE
C             TEST
C
      GUESS = STR*B
      DO 32  I = 1,2C0
C
C             IN THE FUNCTION SUBPROGRAM "STRESS", COMPUTE THE STRESS
C             IN COMPONENT B AT THE 200 TIME INTERVALS (STRB).  USE THE
C             LAST COMPUTED VALUE OF STRB AS THE NEW GUESS AT THE NEXT
C             TIME INTERVAL.
C
      STRB(I) = STRESS(GUESS,TZ(I),KA,KB,PA,PB,BA,BB,NA,NB,RA,RB,MA,MB,
     1CA,CB,QA,QB,A,B,STR)
      IF(STRB(I).EQ.50000.)  GO TO 774
      J = J + 1
   32 GUESS = STRB(I)
      DO 33  I = 1,200
C
C             CALCULATE THE STRESS IN COMPONENT A (STRA).
C
      STRA(I) = (STR - B*STRB(I))/A
C
C             USE FUNCTION SUBPROGRAM "STRAIN" TO COMPUTE THE FOLLOWING:
C                ECOM = PREDICTED CREEP STRAIN IN COMPOSITE WHEN
C                       LOADED AT STRESS LEVEL EQUAL "STR"
C                EA = COMPUTED STRAIN IN COMPONENT A ALONE WHEN
C                     LOADED AT STRESS EQUAL TO "STR".
C                EB = COMPUTED STRAIN IN COMPONENT B ALONE WHEN LOADED
C                     AT STRESS EQUAL TO "STR".
C
      ECOM(I) = STRAIN(STRB(I),TZ(I),KB,PB,BB,NB,RB,MB,CB,QB)
      EA(I) = STRAIN(STR,TZ(I),KA,PA,BA,NA,RA,MA,CA,QA)
      EB(I) = STRAIN(STR,TZ(I),KB,PB,BB,NB,RB,MB,CB,QB)
C
C             CHANGE THE SIGNS FOR CLARITY ON GRAPHICAL PRESENTATION.
C
      EAM(I) = -EA(I)
      EBM(I) = -EB(I)
   33 ECOMM(I) = -ECOM(I)
C
C             CHANGE EXPERIMENTAL DEFORMATION IN MILS TO STRAIN (IN/IN).
C
      DO 34  I = 1,NPTS
      EMEAS(I) = EMEAS(I)/(1000.*GL)
   34 EMEASM(I) = -EMEAS(I)
C
C             THE NEXT TWELVE STATEMENTS USE SUBROUTINE DRAW TO PLOT THE
C             COMPUTED CREEP CURVE FOR THE INDIVIDUAL COMPONENTS AND
C             FOR THE COMPOSITE.
C
      LABB = LABEL(1)
      CALL DRAW(200,EAM,TZ,1,0,LABB,ITITLE,EPSKAL,TSKAL,0,8,2,2,8,15,1,
     1LASTA)
      PRINT 40,LASTA
   40 FORMAT(//,10X,' LASTA = ',I2,//)
      LABB = LABEL(2)
      CALL DRAW(200,EBM,TZ,2,0,LABB,ITITLE,EPSKAL,TSKAL,0,8,2,2,8,15,1,
     1LASTB)
      PRINT 41,LASTB
   41 FORMAT(//,10X, ' LASTB = ',I2,//)
      LABB = LABEL(3)
      CALL DRAW(200,ECOMM,TZ,2,0,LABB,ITITLE,EPSKAL,TSKAL,0,8,2,2,8,15,1
     1,LASTM)
      PRINT 42,LASTM
   42 FORMAT(//,10X,' LASTM = ',I2,//)
```

```
C
C                THE NEXT TWENTY STATEMENTS ARE USEO TO SELECT 30 EXPERI-
C                MENTAL OATA POINTS, THE MAXIMUM ALLOWEO IN SU8ROUTINE
C                ORAW.  THE SELECTED EXPERIMENTAL POINTS ARE SUPFRIMPOSEO
C                ON THE PREOICTED CREEP CURVE.
C
      IF(NPTS.GT.59)  GO TO 776
      IF(NPTS.GT.30)  GO TO 50
      IPTS = NPTS
      DO 49  I = 1,IPTS
      EG(I) = EMEASMII)
   49 TG(I) = TMEAS(I)
      GO TO 53
   50 JF = NPTS - 30
      DO 51  I = 1,JF
      EGII) = EMEASM(2*I)
   51 TG(I) = TMEAS(2*I)
      JF2 = (2*JF) + 1
      DO 52  I = JF2,NPTS
      EG(I-JF) = EMEASM(I)
   52 TG(I-JF) = TMEASII)
      IPTS = 30
   53 CONTINUE
      CALL DRAWIIPTS,EG,TG,3,3,RUN,ITITLE,EPSKAL,TSKAL,D,8,2,2,B,15,1,LA
     1STD)
      PRINT 43,LASTD
   43 FORMATI//,10X,' LASTO = ',I2,//)
C
C                GENERATE 1DD TIMES (TY) COVERING THE FIRST 2DO SECONOS
C                OF THE TEST.  USE FUNCTION SU8PROGRAM "STRESS" TO FIND
C                THE CORRESPONDING STRESSES ID8) IN COMPONENT 8.
C
      FINC = 2./3600.
      TY(1) = FINC
      DO 7D  I = 2,100
   70 TY(I) = TY(I-1) + FINC
      J = 1
      GUESS = STR*8
      DO 73  I = 1,100
      DB(I) = STRESS(GUESS,TY(I),KA,K8,PA,PB,8A,BB,NA,NB,RA,RB,MA,MB,CA,
     1C8,OA,08,A,8,STR)
      IF(C8(I).EQ.50D00.)  GO TO 774
      J = J + 1
   73 GUESS = DB(I)
C
C                CALCULATE THE STRESS IN COMPONENT A (OA) ANO THF
C                LOAD FRACTIONS:
C                   FRACTA = FRACTION OF THE LOAO CARRIED BY COMPONENT A.
C                   FRACT8 = FRACTION OF THE LOAD CARRIED BY COMPONENT B.
C
      DO 74  I = 1,1DD
      DA(I) = ISTR - B*DBII))/A
      FRACTA(I) = -(DA(I)*A)/STR
      FRACTB(I) = -(DBII)*8)/STR
   74 TS(I) = TY(I)*3600.
      LABB = LA8ELI1)
      KTITLE(12) = RUN
C
C                USE SUBROUTINE DRAW TO PLOT THE LOAD DISTRIBUTION DURING
C                THE FIRST 200 SECONDS.
C
      CALL DRAW(1DD,FRACTA,TS,1,0,LA88,KTITLE,0.2,20. ,0,6,2,2,6,15,1,LA
     1STA)
      PRINT 40,LASTA
      LABB = LABELI2)
      CALL DRAW(100,FRACTB,TS,3,0,LABB,KTITLE,0.2,20. ,0,6,2,2,6,15,1,LA
     1STB)
      PRINT 41,LAST8
      DO 75  I = 1,1DO
      FRACTA(I) = -FRACTA(I)
   75 FRACTB(I) = -FRACTB(I)
      J = 1
C
C                CALCULATE THE STRESS (SIGG) IN COMPONENT 8 (USING FUNCTION
C                SUBPROGRAM "STRESS") CORRESPONDING TO THE EXPERIMENTAL
C                TIMES (TMEAS).  USE FUNCTION SUBPROGRAM "STRAIN" TO FINO
C                THE PREOICTED STRAIN (ECALC) CORRESPONDING TO THE DATA
C                POINTS SO A TABULATED COMPARISON MAY BE MADE.
C
      GUESS = STRB(1)
      DO 36  I = 1,NPTS
      SIGG(I) = STRESS(GUESS,TMEASII),KA,K8,PA,PB,BA,88,NA,NB,RA,RB,MA,
     1MB,CA,CB,QA,QB,A,B,STR)
      IF(SIGG(I).EQ.50000.)  GO TO 774
      ECALC(I) = STRAIN(SIGG(I),TMEAS(I),K8,P8,BB,NB,R8,M8,CB,OB)
      J = J+1
   36 GUESS = SIGG(I)
C
C                COMPUTE THE PERCENT ERROR IN PREDICTED STRAIN
C
      DO 37  I = 1,NPTS
   37 ERR(I) = ((EMEAS(I) - ECALC(I))/EMEAS(I))*1DO.
C
C                FINO THE LOAD FRACTIONS IFA,FB) AND THE STRESSES AT 25
C                SELECTED TIMES (TQ) COVERING THE DURATION OF THE TEST.
C
      OO 38  I = 1,25
      FA(I) = (STRAII*8-7)*A)/STR
      FB(I) = (STRB(I*8-7)*8)/STR
      TO(I) = TZ(I*8-7)
      SA(I) = STRA(I*B-7)
   3B SBII) = STRB(I*B-7)
C
C                PRINT OUT THE COMPUTED RESULTS IN TABULAR FORM.
C
      PRINT 20, RUN
   2D FORMAT(/////,10X,' COMPARISON OF THE CALCULATED AND EXPERIMENTAL V
     1ALUES OF CREEP STRAIN',/,10X,' FOR COMPOSITE RUN',A8,' IS GIVEN BE
     2LOW:',///)
      PRINT 21
   21 FORMAT(10X,' TIME,HOURS       EXPTL. STRAIN         CALC. STRAIN
     1    PCT. ERROR',//)
      PRINT 22,(TMEAS(I),EMEAS(I),ECALC(I),ERR(I), I = 1,NPTS)
   22 FORMAT(1HO,F20.5,F1B.6,F20.6,F15.2)
      PRINT 23
   23 FORMAT(/////,10X,' THE CALCULATED STRESSES CARRIED BY EACH MATERIA
     1L OF THE COMPOSITE AND THE CORRESPONDING',/,10X,' LOAD FRACTIONS A
     2RE GIVEN BELOW FOR 25 SELECTED TIMES COVERING THE DURATION OF THE
     3TEST',///)
      PRINT 24
   24 FORMAT(7X,' TIME,           STRESS IN          STRESS IN      LOAD FRACT
     1ION    LOAD FRACTION',/,7X,' HOURS        MATERIAL "A"      MATERIAL
     2"8"    CARRIED BY "A"   CARRIED BY "B"',//)
      PRINT 25,(TQ(I),SA(I),SB(I),FA(I),FB(I),  I = 1,25)
   25 FORMAT(1HO,F13.5,F15.1,F17.1,F16.3,F17.3)
      PRINT 26
   26 FDRMAT(/////,10X,' THE LOAO FRACTIONS ANO STRESSES CARRIED BY EACH
     1',/,1DX,' COMPONENT DURING THE INITIAL PART OF THE TEST ARE LISTED
     2 BELOW:',///)
      PRINT 27
   27 FORMAT(7X,' TIME,           STRESS IN          STRESS IN      LOAD FRACT
     1ION    LOAD FRACTION',/,7X,' SECONOS      MATERIAL "A"      MATERIAL
     2"B"    CARRIED BY "A"   CARRIEO BY "B"',//)
      PRINT 28,ITS(I),OA(I),DB(I),FRACTA(I),FRACT8(I),  I = 1,100)
   28 FORMAT(1HO,F13.0,F15.1,F17.1,F16.3,F17.3)
      GO TO 776
  774 PRINT 775,J
  775 FORMATI////,2DX,' CONVERGENCE FAILURE, J =',I3,///)
  776 CONTINUE
      END
```

```
FORTRAN IV G LEVEL 1, MOD 1           STRESS              DATE = 68139           08/19/41                  PAGE 0001

      FUNCTION STRESS(SIGB,T,KA,KB,PA,PB,BA,BB,NA,NB,RA,RB,MA,MB,CA,CB,
C
C              THIS SUBPROGRAM USES A NEWTON-RAPHSON ITERATION TO
C              COMPUTE THE STRESS IN COMPONENT B (SIGB) FOR A GIVEN
C              TIME (T).  IN THE FUNCTION (FUNCT) THE ONLY UNKNOWN IS
C              SIGB.  THE FUNCTION IS FORMED BY EQUATING THE STRAIN IN
C              EACH COMPONENT.  THE STRAIN IN EACH COMPONENT IS GIVEN BY
C              THE PREVIOUSLY DETERMINED EMPIRICAL CREEP EQUATION,
C              WHOSE PARAMETERS ARE READ IN AS ARGUMENTS OF THE
C              SUBPROGRAM.  THE FUNCTION IS FORMED BY:
C                   FUNCT = STRAIN IN A - STRAIN IN B = 0.
C              THE DERIVATIVE (DERIV) OF THE FUNCTION IS TAKEN
C              ANALYTICALLY.
C
     1DA,DB,A,B,STR)
      REAL KA,KB,NA,NB,MA,MB
      L = 0
  101 SIGBOL = SIGB
      SIGA = (STR - B*SIGB)/A
      TERM1 = KA*(SIGA**NA) - KB*(SIGB**NB) + BA*(SIGA**RA)*T - BB*(SIGB
     1**RB)*T
      PWRA = CA*(T**DA)
      IF(PWRA-50.) 82,82,81
   81 GGA = 1.0
      GO TO 83
   82 GGA = 1. - EXP(-PWRA)
   83 PWRB = CB*(T**DB)
      IF(PWRB-50.) 85,85,84
   84 GGB = 1.0
      GO TO 86
   85 GGB = 1. - EXP(-PWRB)
   86 FUNCT = TERM1 + PA*(SIGA**MA)*GGA - PB*(SIGB**MB)*GGB
      H = -B/A
      DERIV = H*(NA*KA*(SIGA**(NA-1.)) + RA*BA*(SIGA**(RA-1.))*T + MA*PA
     1*(SIGA**(MA-1.))*GGA) - (NB*KB*(SIGB**(NB-1.)) + RB*BB*(SIGB**(RB-
     21.))*T + MB*PB*(SIGB**(MB-1.))*GGB)
      SIGB = SIGB - (FUNCT/DERIV)
      DIFF = ABS(SIGB - SIGBOL)
      L = L+1
C
C              IF THE ROOT OF FUNCT (= SIGB) IS NOT FOUND IN TWENTY
C              ITERATIONS, ASSUME CONVERGENCE FAILURE.
C
      IF(L.GT.20) GO TO 94
      IF(DIFF - 0.5) 93,93,101
   93 STRESS = SIGB
      RETURN
   94 STRESS = 50000.
      RETURN
      END
```

```
FORTRAN IV G LEVEL 1, MOD 1           STRAIN              DATE = 68139           08/19/41                  PAGE 0001

      FUNCTION STRAIN(S,Z,K,P,B,N,R,M,C,D)
      REAL K,M,N
C
C              THIS SUBPROGRAM USES THE CREEP EQUATION, WHOSE PARAMETERS
C              ARE READ IN AS SUBPROGRAM ARGUMENTS, TO COMPUTE THE STRAIN
C              FOR A GIVEN STRESS(S) AND A GIVEN TIME(Z).
C
      POWER = C*(Z**D)
      IF(POWER -50.) 62,62,61
   61 PROD = 1.0
      GO TO 63
   62 PROD = 1.-EXP(-POWER)
   63 STRAIN = K*(S**N) + B*(S**R)*Z + P*(S**M)*PROD
      RETURN
      END
```

TABULATED OUTPUT FROM
PROGRAM COMCUR
FOR PE/PP COMPOSITE
STRESS = 1000PSI

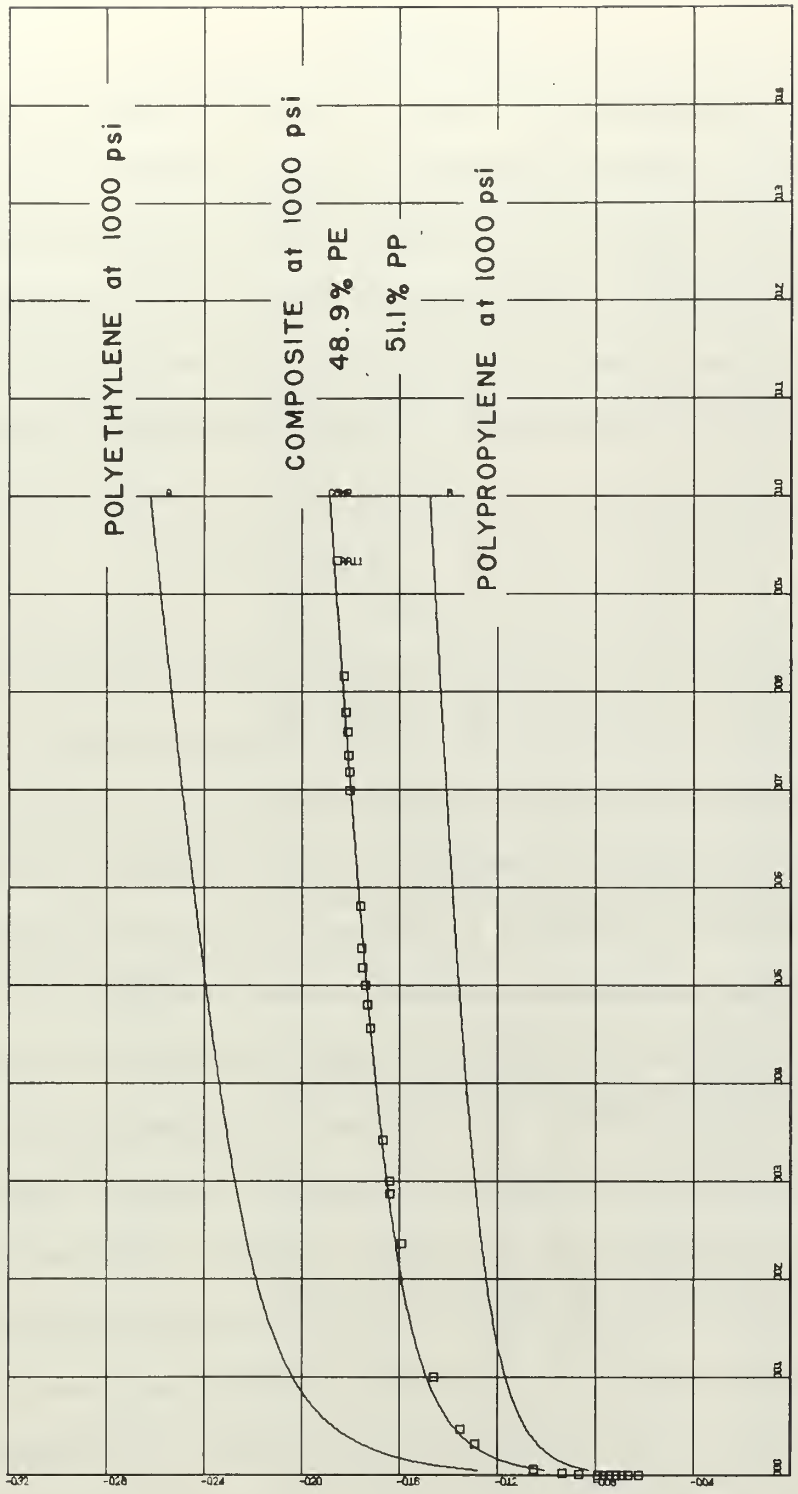
```
COMPARISON OF THE CALCULATED AND EXPERIMENTAL VALUES OF CREEP STRAIN
FOR COMPOSITE RUN    AA11 IS GIVEN BELOW:

TIME,HOURS       EXPTL. STRAIN         CALC. STRAIN        PCT. ERROR

   0.00056          0.005500              0.004749            13.66



   0.00139          0.006200              0.005013            19.15
   0.00278          0.006400              0.005274            17.60
   0.00417          0.006650              0.005456            17.96
   0.00556          0.006800              0.005599            17.66
   0.00694          0.006900              0.005719            17.12
   0.00833          0.007000              0.005823            16.81
   0.01111          0.007150              0.006001            16.08
   0.01388          0.007300              0.006148            15.78
   0.01667          0.007400              0.006277            15.18
   0.02222          0.007550              0.006492            14.01
   0.02777          0.007650              0.006672            12.79
   0.03330          0.007750              0.006826            11.92
   0.05000          0.007900              0.007199             8.87
   0.08333          0.008350              0.007724             7.50
   0.13333          0.008650              0.008264             4.47
   0.21667          0.009050              0.008879             1.89
   0.30000          0.009350              0.009323             0.29
   0.38333          0.010050              0.009674             3.74
   0.63333          0.010500              0.010430             0.66
   1.80000          0.011800              0.012122            -2.73
   3.30000          0.012900              0.013128            -1.77
   3.80000          0.013050              0.013361            -2.38
   4.80000          0.013500              0.013741            -1.79
   6.03330          0.013900              0.014108            -1.49
  10.08300          0.014600              0.014904            -2.09
  21.64999          0.015900              0.016035            -0.85
  23.64999          0.015900              0.016165            -1.67
  27.16699          0.016250              0.016371            -0.74
  28.79999          0.016400              0.016459            -0.36
  30.04999          0.016400              0.016523            -0.75
  34.29999          0.016650              0.016727            -0.46
  45.63300          0.017200              0.017193             0.04
  48.01669          0.017300              0.017281             0.11
  50.01669          0.017400              0.017353             0.27
  51.86699          0.017500              0.017418             0.47
  53.84999          0.017550              0.017487             0.36
  58.20000          0.017600              0.017633            -0.19
  69.93300          0.018000              0.018003            -0.02
  71.83299          0.018000              0.018061            -0.34
  73.59999          0.018050              0.018114            -0.36
  75.93300          0.018100              0.018184            -0.46
  77.96700          0.018200              0.018243            -0.24
  81.66699          0.018250              0.018351            -0.55
  93.43300          0.018550              0.018685            -0.73
```

```
THE CALCULATED STRESSES CARRIED BY EACH MATERIAL OF THE COMPOSITE AND THE CORRESPONDING
LOAD FRACTIONS ARE GIVEN BELOW FOR 25 SELECTED TIMES COVERING THE DURATION OF THE TEST

  TIME,        STRESS IN         STRESS IN       LOAD FRACTION     LOAD FRACTION
  HOURS      MATERIAL "A"      MATERIAL "B"     CARRIED BY "A"    CARRIED BY "B"

  0.50000        852.6            1141.0            0.417             0.583
  4.50000        824.9            1167.5            0.403             0.597
  8.50000        821.2            1171.1            0.402             0.598
 12.50000        819.9            1172.4            0.401             0.599
 16.50000        819.3            1172.9            0.401             0.599
 20.50000        819.1            1173.1            0.401             0.599
 24.50000        818.9            1173.3            0.400             0.600
 28.50000        818.9            1173.3            0.400             0.600
 32.50000        818.8            1173.4            0.400             0.600
 36.50000        818.7            1173.5            0.400             0.600
 40.50000        818.6            1173.6            0.400             0.600
 44.50000        818.5            1173.7            0.400             0.600
 48.50000        818.4            1173.8            0.400             0.600
 52.50000        818.3            1173.9            0.400             0.600
 56.50000        818.1            1174.0            0.400             0.600
 60.50000        818.0            1174.2            0.400             0.600
 64.50000        817.8            1174.3            0.400             0.600
 68.50000        817.6            1174.5            0.400             0.600
 72.50000        817.4            1174.7            0.400             0.600
 76.50000        817.2            1174.9            0.400             0.600
 80.50000        817.0            1175.1            0.400             0.600
 84.50000        816.8            1175.3            0.399             0.601
 88.50000        816.6            1175.5            0.399             0.601
 92.50000        816.4            1175.7            0.399             0.601
 96.50000        816.1            1176.0            0.399             0.601
```

A = PE
B = PP

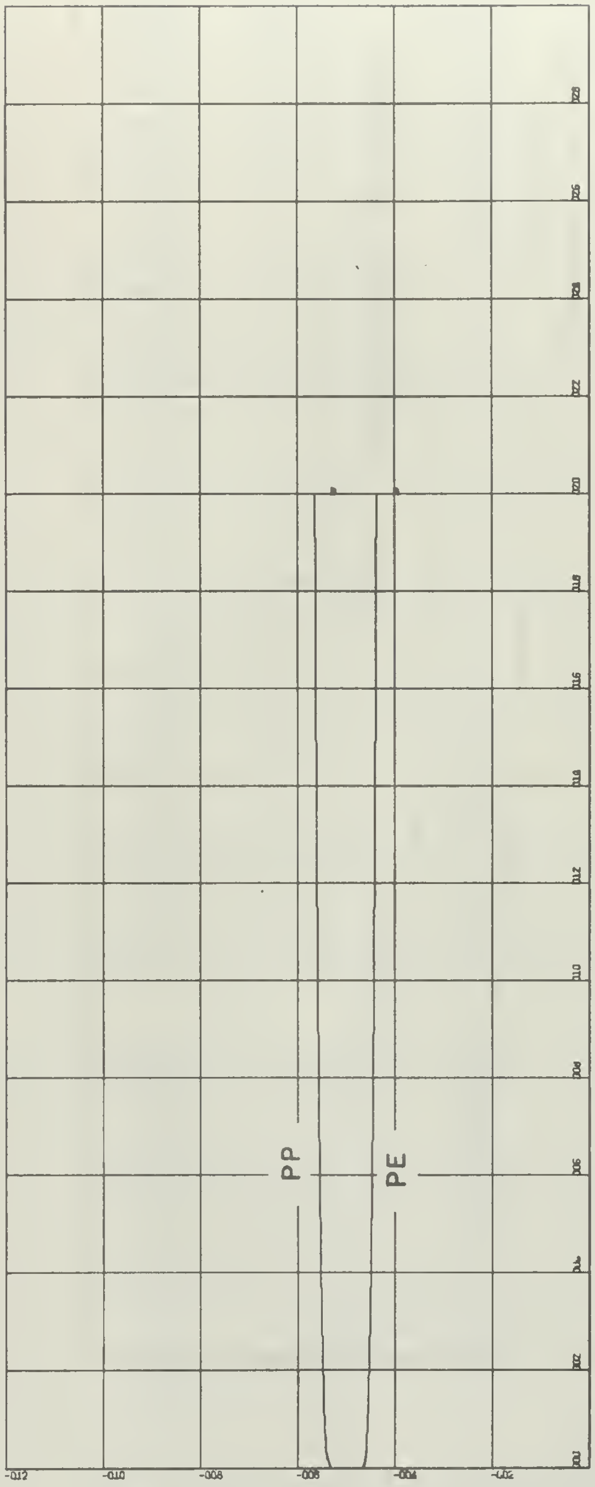
```
THE LOAD FRACTIONS AND STRESSES CARRIED BY EACH
COMPONENT DURING THE INITIAL PART OF THE TEST ARE LISTED BELOW:

  TIME,        STRESS IN         STRESS IN       LOAD FRACTION     LOAD FRACTION
 SECONDS     MATERIAL "A"      MATERIAL "B"     CARRIED BY "A"    CARRIED BY "B"

     2.          939.9            1057.5            0.460             0.540
     4.          936.5            1060.7            0.458             0.542
     6.          933.9            1063.2            0.457             0.543
     8.          931.8            1065.2            0.456             0.544
    10.          930.0            1067.0            0.455             0.545
    12.          928.4            1068.5            0.454             0.546
    14.          927.0            1069.9            0.453             0.547
    16.          925.7            1071.1            0.453             0.547
    18.          924.5            1072.3            0.452             0.548
    20.          923.3            1073.4            0.452             0.548
    22.          922.3            1074.3            0.451             0.549
    24.          921.3            1075.3            0.451             0.549
    26.          920.4            1076.1            0.450             0.550
    28.          919.6            1077.0            0.450             0.550
    30.          918.7            1077.8            0.449             0.551
    32.          918.0            1078.5            0.449             0.551
    34.          917.2            1079.2            0.449             0.551
    36.          916.5            1079.9            0.448             0.552
    38.          915.8            1080.6            0.448             0.552
    40.          915.2            1081.2            0.448             0.552
    42.          914.5            1081.8            0.447             0.553
    44.          913.9            1082.4            0.447             0.553
    46.          913.3            1083.0            0.447             0.553
    48.          912.7            1083.5            0.446             0.554
    50.          912.2            1084.0            0.446             0.554
    52.          911.7            1084.5            0.446             0.554
    54.          911.1            1085.0            0.446             0.554
    56.          910.6            1085.5            0.445             0.555
    58.          910.1            1086.0            0.445             0.555
    60.          909.7            1086.4            0.445             0.555
```

62.	909.2	1086.9	0.445	0.555
64.	908.8	1087.3	0.444	0.556
66.	908.3	1087.7	0.444	0.556
68.	907.9	1088.2	0.444	0.556
70.	907.5	1088.6	0.444	0.556
72.	907.0	1088.9	0.444	0.556
74.	906.6	1089.3	0.443	0.557
76.	906.3	1089.7	0.443	0.557
78.	905.9	1090.1	0.443	0.557
80.	905.5	1090.4	0.443	0.557
82.	905.1	1090.8	0.443	0.557
84.	904.8	1091.1	0.442	0.558
86.	904.4	1091.5	0.442	0.558
88.	904.1	1091.8	0.442	0.558
90.	903.7	1092.1	0.442	0.558
92.	903.4	1092.5	0.442	0.558
94.	903.1	1092.8	0.442	0.558
96.	902.7	1093.1	0.441	0.559
98.	902.4	1093.4	0.441	0.559
100.	902.1	1093.7	0.441	0.559
102.	901.8	1094.0	0.441	0.559
104.	901.5	1094.3	0.441	0.559
106.	901.2	1094.5	0.441	0.559
108.	900.9	1094.8	0.441	0.559
110.	900.6	1095.1	0.440	0.560
112.	900.3	1095.4	0.440	0.560
114.	900.1	1095.6	0.440	0.560
116.	899.8	1095.9	0.440	0.560
118.	899.5	1096.2	0.440	0.560
120.	899.2	1096.4	0.440	0.560
122.	899.0	1096.7	0.440	0.560
124.	898.7	1096.9	0.439	0.561
126.	898.5	1097.2	0.439	0.561
128.	898.2	1097.4	0.439	0.561
130.	898.0	1097.6	0.439	0.561
132.	897.7	1097.9	0.439	0.561
134.	897.5	1098.1	0.439	0.561
136.	897.2	1098.3	0.439	0.561
138.	897.0	1098.6	0.439	0.561
140.	896.8	1098.8	0.439	0.561
142.	896.5	1099.0	0.438	0.562
144.	896.3	1099.2	0.438	0.562
146.	896.1	1099.4	0.438	0.562
148.	895.9	1099.7	0.438	0.562
150.	895.6	1099.9	0.438	0.562
152.	895.4	1100.1	0.438	0.562
154.	895.2	1100.3	0.438	0.562
156.	895.0	1100.5	0.438	0.562
158.	894.8	1100.7	0.438	0.562
160.	894.6	1100.9	0.437	0.563
162.	894.4	1101.1	0.437	0.563
164.	894.2	1101.3	0.437	0.563
166.	894.0	1101.5	0.437	0.563
168.	893.8	1101.6	0.437	0.563
170.	893.6	1101.8	0.437	0.563
172.	893.4	1102.0	0.437	0.563
174.	893.2	1102.2	0.437	0.563
176.	893.0	1102.4	0.437	0.563
178.	892.8	1102.6	0.437	0.563
180.	892.6	1102.7	0.436	0.564
182.	892.4	1102.9	0.436	0.564
184.	892.3	1103.1	0.436	0.564
186.	892.1	1103.3	0.436	0.564
188.	891.9	1103.4	0.436	0.564
190.	891.7	1103.6	0.436	0.564
192.	891.5	1103.8	0.436	0.564
194.	891.4	1104.0	0.436	0.564
196.	891.2	1104.1	0.436	0.564
198.	891.0	1104.3	0.436	0.564
200.	890.9	1104.4	0.436	0.564



X-SCALE=4.00E-03 UNITS INCH.
Y-SCALE=1.00E+01 UNITS INCH.
HERING,BOX 21, COMPARISON OF CALCULATED CREEP CURVE AND ACTUAL DATA FOR COMPOSITE AB

CALIFORNIA COMPUTER PRODUCTS, INC. ANAHEIM, CALIFORNIA



X-SCALE=2.00E-01 UNITS INCH.
Y-SCALE=2.00E+01 UNITS INCH.
HERING,BOX 21, LOAD FRACTIONS IN EACH COMPONENT
OF COMPOSITE AB VS. TIME -- CREEP TEST AA11

CALIFORNIA COMPUTER PRODUCTS, INC. ANAHEIM CALIFORNIA CHART NO

APPENDIX D

PREDICTION OF STRESS RELAXATION BEHAVIOR FROM EMPIRICAL CREEP EQUATION

A stress relaxation test is performed by rapidly deforming a material to a predetermined strain then observing the decrease in stress as a function of time while this strain is maintained. A series of such tests were run on the Instron testing machine for the plastics used in this study. The empirical equation developed for creep behavior was used to predict the relaxation curve.

$$\epsilon = K\sigma^N + B\sigma^R \cdot t + P\sigma^M\left(1 - e^{-ct^Q}\right) \tag{D.1}$$

Again the equation of state assumption was made for predicting relaxation behavior. Thus, for a given strain, and a series of times, the corresponding stress may be found by implicitly solving equation (D.1). This was done by a Fortran IV computer program called program RELAX. The input data consisted of the creep parameters previously determined and the initial strain. A Newton-Raphson iteration was used to solve equation (D.1) for stress as a function of time, and these values were tabulated as output. A plotting subroutine available at the NPGS Computer Facility (SUBROUTINE DRAW) was used to display the predicted relaxation curves. Figure 32 illustrates a computed relaxation curve compared with the experimental relaxation curve during the initial part of the test. Figure 33 shows the predicted curve and experimental data over the entire time frame of the experiment. These results are typical of all runs; the same general trend was observed for each plastic, even at low stress levels. The empirical creep equation predicts a greater decrease in stress than that observed experimentally. Apparently there is a structure factor influence creating irreversibilities that are unaccounted for in the creep equation. The equation of state assumption is not valid -- the stress level depends

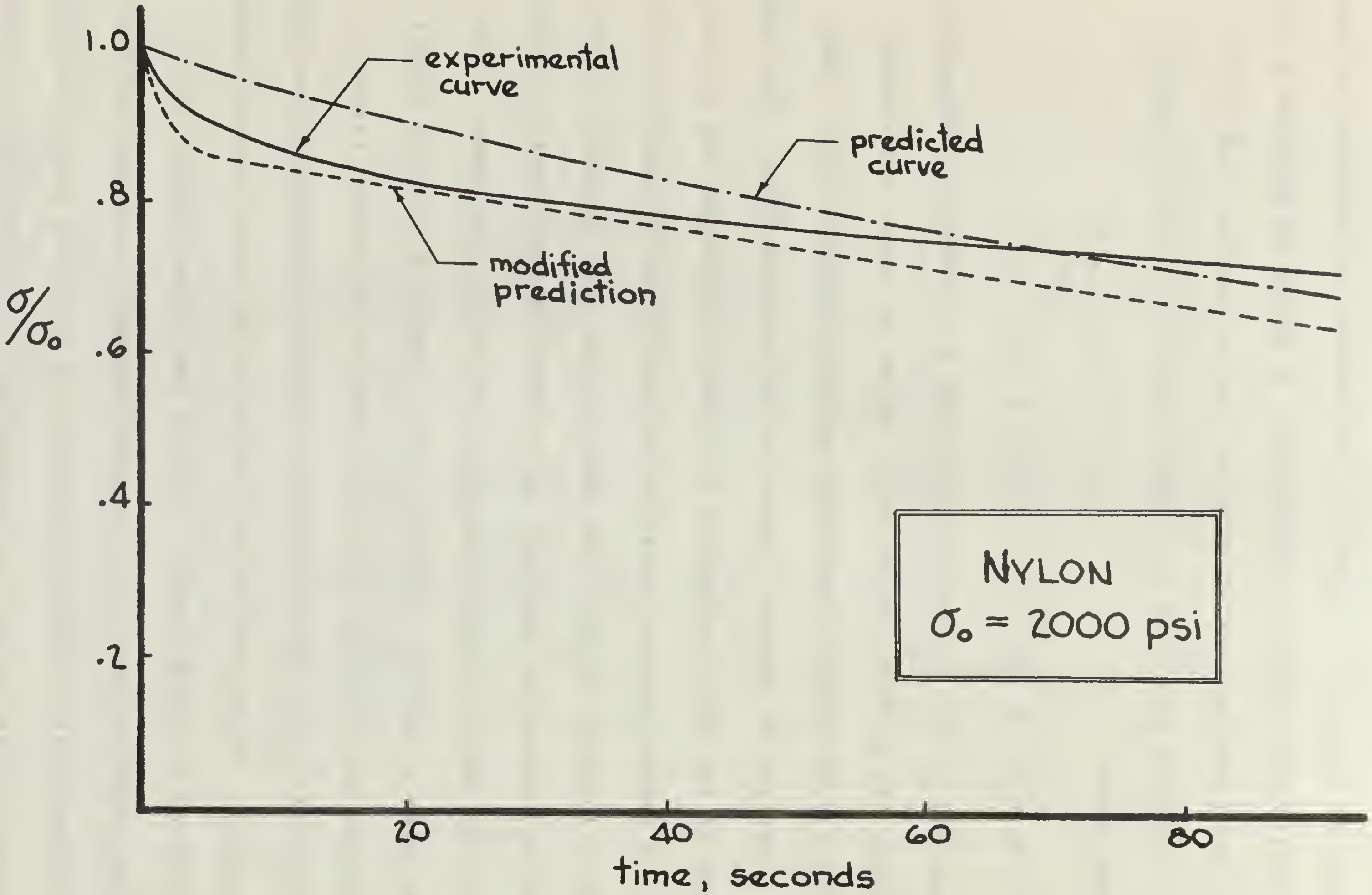


Figure 32

Predicted Short-Term Relaxation Behavior of Nylon

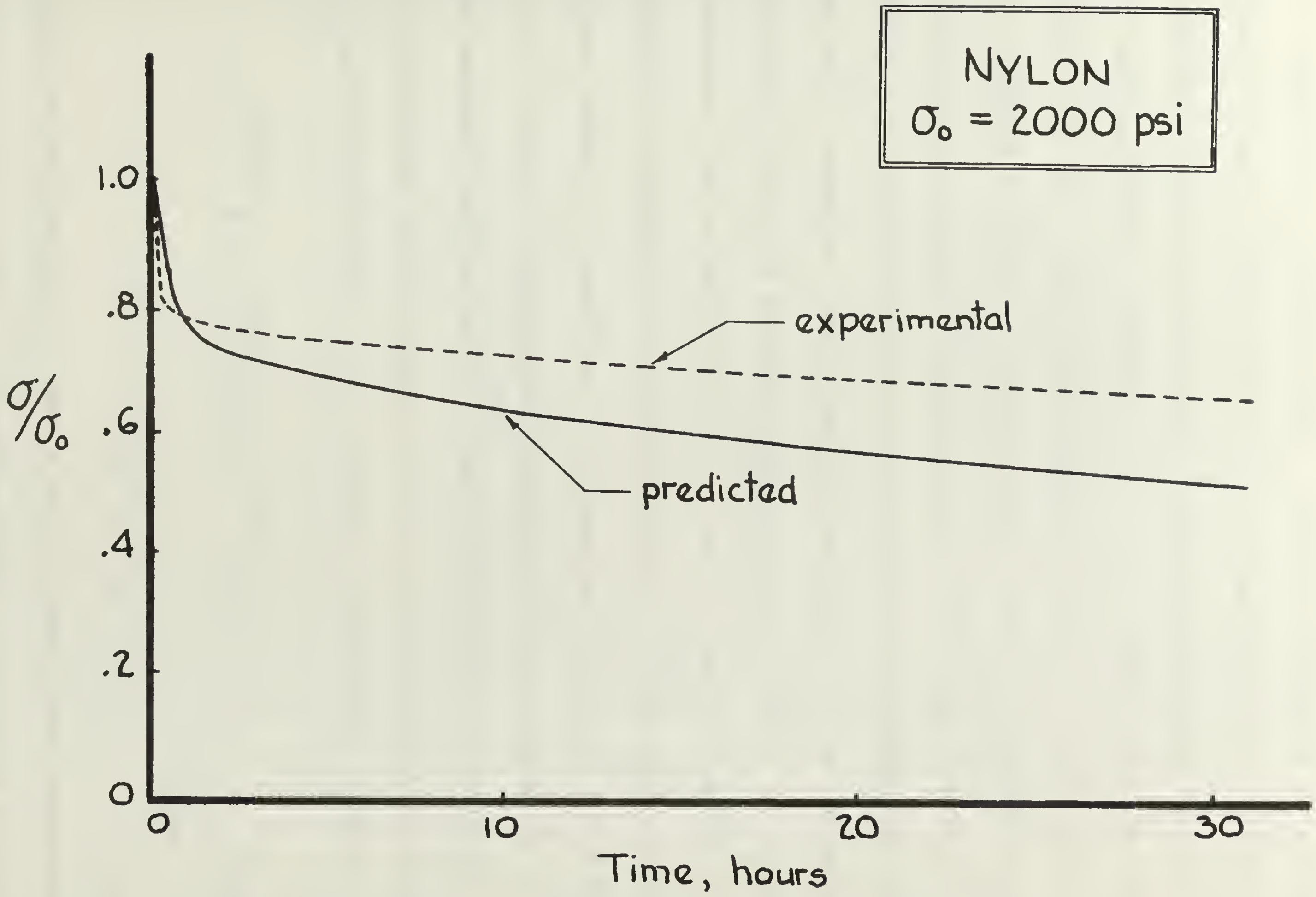


Figure 33

Predicted Long-Term Relaxation Behavior of Nylon Based on Creep Equation on the strain history.

An interesting observation can be made from the experimental data in Figure 32. During the first ten seconds of the test, the stress fell off rapidly in exponential fashion. This indicates the presence of molecular mechanisms with relaxation times on the order of several seconds. The corresponding retardation time in the creep equation is approximately one hour; the predicted curve does not show this sudden decrease at the start of the test. It can be concluded, therefore, that the "instantaneous strain", parameter A_1 in the creep equation, includes a portion of the delayed strain with very short time constants. The time scale of the creep test and the method of extrapolation to determine parameter A_1 prevents the resolution of this rapid but delayed elastic response. From Appendix A, Table 3, it is seen that the stress exponent N for the instantaneous strain is slightly greater than 1. If the delayed strain component combined in the term $K\sigma^N$ could be removed, this exponent would decrease. Although not a necessary consequence of the theory of rubber-like elasticity [2], it appears that the exponent may approach a value of 1, so that under instantaneous loading the plastics behave as ideal Hookean solids.

A modified equation was used with program RELAX, separating the parameter A_1 into an instantaneous response and a rapid delayed elastic response:

(D.2) $$\epsilon = K_1\sigma^N + K_2\sigma^N\left(1 - e^{-t/\tau_a}\right)$$

where

(D.3) $$K = K_1 + K_2$$

The stress coefficients K_1 and K_2 were estimated from the experimental relaxation data and the time constant τ_a taken to be 10 seconds. The predicted relaxation curve then matched the data more closely at short times, but the same underestimate of stress level developed at long times. (See Figure 33).

COMPUTER PROGRAM "RELAX"

```
FORTRAN IV G LEVEL 1, MOD 1            MAIN              DATE = 68125          08/24/38                PAGE 0001

            REAL K,N,M,LABEL/4H    /,LOAD(200)
            REAL*8 ITITLE(12)
            DIMENSION PLAST(5),STR(50),T(200),SIG(200),TA(200),SIGA(200),SIGB(
           1200),TB(200),TH(200),AREA(50)
      C
      C                READ IN THE FOLLOWING DATA
      C                   ITITLE = TITLE TO BE USED ON GRAPHICAL OUTPUT
      C                   PLAST = NAME OF MATERIAL TESTED
      C                   NOAT = NUMBER OF INITIAL STRESS CONDITIONS FOR WHICH
      C                          THE RELAXATION CURVE IS TO BE INVESTIGATED.
      C                   K,C,B,Q
      C                   P,R,N,M = PARAMETERS IN THE EMPIRICAL CREEP EQUATION
      C                             FOR THE MATERIALS TESTED
      C                   STR = ARRAY OF INITIAL STRESS VALUES
      C                  AREA = ARRAY OF CROSS-SECTIONAL AREAS FOR THE EXPERIMENTAL
      C                         RUNS, CORRESPONDING TO THE INITIAL STRESS VALUES.
      C
            READ 13,(ITITLE(I), I = 1,12)
         13 FORMAT(6A8)
            READ 10,(PLAST(I), I = 1,4), NOAT
         10 FORMAT(4A4,I4)
            READ 11, K,B,P,C,Q,N,R,M
         11 FORMAT(3E15.5,5F6.4)
            READ 12,(STR(I), I = 1,NOAT)
         12 FORMAT(8F10.0)
            KCUE = 50
            KCUE = 0
            IF(KCUE.EQ.50)  GO TO 16
            IF(KCUE.EQ.0) GO TO 14
         14 READ 12,(AREA(I), I = 1,NOAT)
         16 CONTINUE
      C
      C                THE DO LOOP "DO 102" COMPUTES THE RELAXATION CURVE FROM
      C                THE EMPIRICAL CREEP EQUATION FOR EACH OF THE INITIAL
      C                STRESS CONDITIONS.
      C
            DO 102  L = 1,NOAT
            S = STR(L)
      C
      C                CALCULATE THE INITIAL STRAIN,EO.  GENERATE 200 TIMES WITH
      C                THE LONGEST ELAPSED TIME APPROXIMATELY 30 HOURS.  STORE
      C                THESE TIMES IN THE ARRAY "T".
      C
            EO = K*(S**N)
            TINC = 1./3600.
            T(1) = TINC
            DO 32  I = 2,60
         32 T(I) = T(I-1) + TINC
            TINC = 10./3600.
            DO 33  I = 61,115
         33 T(I) = T(I-1) + TINC
            TINC = 1./30.
            DO 34  I = 116,141
         34 T(I) = T(I-1) + TINC
            TINC = 0.5
            DO 35  I = 142,199
         35 T(I) = T(I-1) + TINC
            S = 0.95*S
      C
      C                THE DO LOOP "DO 35" USES A NEWTON-RAPHSON ITERATION TO
      C                SOLVE THE EMPIRICAL CREEP EQUATION IMPLICITLY FOR STRESS
      C                AT EACH TIME IN THE ARRAY "T".  THE INITIAL STRAIN IS EO.
      C                THE FIRST ESTIMATE OF THE RELAXED STRESS AT ELAPSED TIME
      C                T(1) = 1 SECOND IS 0.95*S.  THE FIRST ESTIMATES AT LATER
      C                TIMES IS THE RELAXED STRESS AT THE PREVIOUS ELAPSED TIME.
      C                STRESSES COMPUTED FOR THE TIMES T(I) ARE STORED IN THE
      C                ARRAY NAMED "SIG".
      C
            DO 36  I = 1,199
            LC = 0
            DIFF = 40000.
            PWR = C*(T(I)**Q)
            IF(PWR - 50.) 82,82,81
         81 D = 1.0
            GO TO 83
         82 D = 1. - EXP(-PWR)
         83 LC = LC + 1
            SOLD = S
            FUNCT = -EO + K*(S**N) + B*(S**R)*T(I) + P*(S**M)*D
            DERIV = N*K*(S**(N-1.))+R*B*(S**(R-1.))*T(I)+M*P*(S**(M-1.))*D
            S = S - (FUNCT/DERIV)
            DIFF = ABS(SOLD -S)
      C
      C                IF THE STRESS IS NOT FOUND WITHIN 0.1 PSI IN TWENTY
      C                ITERATIONS, ASSUME CONVERGENCE FAILURE.
      C
            IF(LC.GT.20)  GO TO 87
            IF(S.LE.0.0)  S = S/2.
            IF(DIFF - 0.1) 36,83,83
         87 PRINT 88, I,LC
         88 FORMAT(10X,' CONVERGENCE FAILURE, I = ',I2,' LC = ',I2,//)
            S = 0.75*STR(L)
         36 SIG(I) = S
      C
      C                COMPUTE THE "FRACTION OF RELAXATION", EQUAL TO RELAXED
      C                STRESS DIVIDED BY INITIAL STRESS, AND STORE IN THE
      C                ARRAY NAMED "SIGA".
      C
            DO 38  I = 2,199
            TA(I) = T(I-1)
         38 SIGA(I) = SIG(I-1)/STR(L)
            TA(1) = 0.0
            SIGA(1) = 1.0
      C
      C                USE SUBROUTINE DRAW TO PLOT THE PREDICTED RELAXATION CURVE
      C                OF "SIGA VS. ELAPSED TIME".
      C
            CALL DRAW(199,TA,SIGA,0,0,LABEL,ITITLE,5.,0.2,1,1,2,2,9,9,1,LAST)
            PRINT 40, LAST
         40 FORMAT(//,10X,' LAST = ', I2,//)
            DO 39  I = 2,200
            TB(I) = T(I-1)*3600.
            TH(I) = T(I-1)
         39 SIGB(I) = SIG(I-1)
            SIGB(1) = STR(L)
            TB(1) = 0.0
            TH(1) = 0.0
            PRINT 44
         44 FORMAT(1H1)
            IF(KCUE.EQ.50)  GO TO 48
```

```
C
C             COMPUTE THE PREDICTED RELAXEO LOAO BASEO ON SPECIMEN AREA
C             SO THAT EXPERIMENTAL LOAD-TIME OATA MAY BF COMPAREO WITH
C             PRFDICTED LOAD-TIME BEHAVIOR.
C
      OO 31  I = 1,200
   31 LOAO(I) = SIGB(I)*AREA(L)
      PR(NT 47,AREA(L)
   47 FORMAT(///,20X,'THE SPECIMEN AREA IS',F7.5,' SQUARF INCHFS',////)
C
C             PRINT OUT THE PREOICTEO STRESS RELAXATIDN CURVF IN
C             TABULAR FORM.
C
      PRINT 45,(PLAST(I), I = 1,4)
      PRINT 49, (TH(I),T8(I),SIGB(I),LOAD(I), I = 1,200)
   49 FORMAT(1H0,20X,F10.6,13X,F8.0,13X,F8.1,13X,F8.1)
      GO TO 103
   48 CONTINUE
      PRINT 45, (PLAST(I), I = 1,4)
   45 FORMAT(10X,'THE CALCULATEO STRESS RELAXATION BEHAVIOR FOR ',4A4,'
     1IS LISTEO BELOW:',//,20X,'TIME,HOURS',10X,'TIME,SFCONDS',10X,' STR
     2ESS,PSI',10X,' LOAD',//)
      PRINT 46,(TH(I),TB(I),SIGB(I),I = 1,200)
   46 FORMAT(1H0,20X,F10.6,13X,F8.0,13X,F8.1)
  103 CONTINUE
  102 CONTINUE
```

TABULATED OUTPUT FROM "RELAX"

```
          THE SPECIMEN AREA IS0.04992 SQUARE INCHES

THE CALCULATED STRESS RELAXATION BEHAVIOR FOR        NYLON      IS LISTED BELOW:
          TIME,HOURS          TIME,SECONDS            STRESS,PSI           LOAD

            0.0                      0.0               2000.0                   99.8
            0.000278                 1.                1999.1                   99.8
            0.000556                 2.                1998.3                   99.7
            0.000833                 3.                1997.5                   99.7
            0.001111                 4.                1996.7                   99.7
            0.001389                 5.                1996.0                   99.6
            0.001667                 6.                1995.2                   99.6
            0.001944                 7.                1994.5                   99.6
            0.002222                 8.                1993.7                   99.5
            0.002500                 9.                1992.9                   99.5
            0.002778                10.                1992.2                   99.4
            0.003056                11.                1991.5                   99.4
            0.003333                12.                1990.7                   99.4
            0.003611                13.                1990.0                   99.3
            0.003889                14.                1989.3                   99.3
            0.004167                15.                1988.5                   99.3
            0.004444                16.                1987.8                   99.2
            0.004722                17.                1987.1                   99.2
            0.005000                18.                1986.4                   99.2
            0.005278                19.                1985.7                   99.1
            0.005556                20.                1984.9                   99.1
            0.005833                21.                1984.2                   99.0
            0.006111                22.                1983.5                   99.0
            0.006389                23.                1982.8                   99.0
            0.006667                24.                1982.1                   98.9
            0.006944                25.                1981.4                   98.9
            0.007222                26.                1980.7                   98.9
            0.007500                27.                1980.0                   98.8
            0.007778                28.                1979.3                   98.8
            0.008056                29.                1978.6                   98.8
            0.008333                30.                1977.9                   98.7
            0.008611                31.                1977.2                   98.7
            0.008889                32.                1976.5                   98.7
            0.009167                33.                1975.8                   98.6
            0.009444                34.                1975.2                   98.6
            0.009722                35.                1974.5                   98.6
            0.010000                36.                1973.8                   98.5
            0.010278                37.                1973.1                   98.5
            0.010556                38.                1972.4                   98.5
            0.010833                39.                1971.7                   98.4
            0.011111                40.                1971.1                   98.4
            0.011389                41.                1970.4                   98.4
            0.011667                42.                1969.7                   98.3
            0.011944                43.                1969.0                   98.3
            0.012222                44.                1968.4                   98.3
            0.012500                45.                1967.7                   98.2
            0.012778                46.                1967.0                   98.2
            0.013056                47.                1966.4                   98.2
            0.013333                48.                1965.7                   98.1
            0.013611                49.                1965.0                   98.1
            0.013889                50.                1964.4                   98.1
            0.014167                51.                1963.7                   98.0
            0.014444                52.                1963.1                   98.0
            0.014722                53.                1962.4                   98.0
            0.015000                54.                1961.7                   97.9
            0.015278                55.                1961.1                   97.9
            0.015555                56.                1960.4                   97.9
            0.015833                57.                1959.8                   97.8
            0.016111                58.                1959.1                   97.8
            0.016389                59.                1958.5                   97.8
            0.016667                60.                1957.8                   97.7
            0.019444                70.                1951.4                   97.4
            0.022222                80.                1945.0                   97.1
```

0.025000	90.	1938.8	96.8
0.027778	100.	1932.7	96.5
0.030555	110.	1926.7	96.2
0.033333	120.	1920.7	95.9
0.036111	130.	1914.8	95.6
0.038889	140.	1909.1	95.3
0.041667	150.	1903.3	95.0
0.044444	160.	1897.7	94.7
0.047222	170.	1892.2	94.4
0.050000	180.	1886.7	94.2
0.052778	190.	1881.2	93.9
0.055555	200.	1875.9	93.6
0.058333	210.	1870.6	93.4
0.061111	220.	1865.3	93.1
0.063889	230.	1860.2	92.9
0.066667	240.	1855.1	92.6
0.069444	250.	1850.0	92.3
0.072222	260.	1845.0	92.1
0.075000	270.	1840.1	91.8
0.077778	280.	1835.2	91.6
0.080555	290.	1830.3	91.4
0.083333	300.	1825.5	91.1
0.086111	310.	1820.8	90.9
0.088889	320.	1816.1	90.7
0.091666	330.	1811.5	90.4
0.094444	340.	1806.9	90.2
0.097222	350.	1802.3	90.0
0.100000	360.	1797.8	89.7
0.102777	370.	1793.4	89.5
0.105555	380.	1789.0	89.3
0.108333	390.	1784.6	89.1
0.111111	400.	1780.2	88.9
0.113888	410.	1776.0	88.6
0.116666	420.	1771.7	88.4
0.119444	430.	1767.5	88.2
0.122222	440.	1763.3	88.0
0.124999	450.	1759.2	87.8
0.127777	460.	1755.1	87.6
0.130555	470.	1751.0	87.4
0.133333	480.	1747.0	87.2
0.136110	490.	1743.0	87.0
0.138888	500.	1739.1	86.8
0.141666	510.	1735.1	86.6
0.144444	520.	1731.3	86.4
0.147221	530.	1727.4	86.2
0.149999	540.	1723.6	86.0
0.152777	550.	1719.8	85.8
0.155555	560.	1716.1	85.7
0.158332	570.	1712.3	85.5
0.161110	580.	1708.6	85.3
0.163888	590.	1705.0	85.1
0.166666	600.	1701.3	84.9
0.169443	610.	1697.7	84.7
0.202777	730.	1656.7	82.7
0.236110	850.	1619.1	80.8
0.269443	970.	1584.6	79.1
0.302777	1090.	1552.8	77.5
0.336110	1210.	1523.2	76.0
0.369443	1330.	1495.7	74.7
0.402777	1450.	1469.9	73.4
0.436110	1570.	1445.8	72.2
0.469443	1690.	1423.1	71.0
0.502777	1810.	1401.7	70.0
0.536110	1930.	1381.5	69.0
0.569443	2050.	1362.4	68.0
0.602776	2170.	1344.3	67.1
0.636110	2290.	1327.1	66.2
0.669443	2410.	1310.7	65.4
0.702776	2530.	1295.0	64.6
0.736110	2650.	1280.1	63.9
0.769443	2770.	1265.9	63.2
0.802776	2890.	1252.2	62.5
0.836110	3010.	1239.2	61.9

```
          0.869443          3130.          1226.6          61.2
          0.902776          3250.          1214.6          60.6
          0.936109          3370.          1203.0          60.0
          0.969443          3490.          1191.9          59.5
          1.002775          3610.          1181.2          59.0
          1.036108          3730.          1170.8          58.4
          1.536108          5530.          1050.9          52.5
          2.036108          7330.           972.3          48.5
          2.536108          9130.           916.7          45.8
          3.036108         10930.           875.3          43.7
          3.536108         12730.           843.5          42.1
          4.036108         14530.           818.4          40.9
          4.536108         16330.           798.2          39.8
          5.036108         18130.           781.7          39.0
          5.536108         19930.           768.2          38.3
          6.036108         21730.           756.8          37.8
          6.536108         23530.           747.3          37.3
          7.036108         25330.           739.3          36.9
          7.536108         27130.           732.4          36.6
          8.036108         28930.           726.6          36.3
          8.536108         30730.           721.5          36.0
          9.036108         32530.           717.2          35.8
          9.536108         34330.           713.5          35.6
         10.036108         36130.           710.2          35.5
         10.536108         37930.           707.4          35.3
         11.036108         39730.           704.9          35.2
         11.536108         41530.           702.7          35.1
         12.036108         43330.           700.9          35.0
         12.536108         45130.           699.2          34.9
         13.036108         46930.           697.7          34.8
         13.536108         48730.           696.4          34.8
         14.036108         50530.           695.2          34.7
         14.536108         52330.           694.2          34.7
         15.036108         54130.           693.3          34.6
         15.536108         55930.           692.5          34.6
         16.036102         57730.           691.7          34.5
         16.536102         59530.           691.1          34.5
         17.036102         61330.           690.5          34.5
         17.536102         63130.           689.9          34.4
         18.036102         64930.           689.4          34.4
         18.536102         66730.           689.0          34.4
         19.036102         68530.           688.6          34.4
         19.536102         70330.           688.2          34.4
         20.036102         72130.           687.8          34.3
         20.536102         73930.           687.5          34.3
         21.036102         75730.           687.2          34.3
         21.536102         77530.           686.9          34.3
         22.036102         79330.           686.6          34.3
         22.536102         81130.           686.4          34.3
         23.036102         82930.           686.1          34.2
         23.536102         84730.           685.9          34.2
         24.036102         86530.           685.7          34.2
         24.536102         88330.           685.5          34.2
         25.036102         90130.           685.2          34.2
         25.536102         91930.           685.0          34.2
         26.036102         93730.           684.8          34.2
         26.536102         95530.           684.7          34.2
         27.036102         97330.           684.5          34.2
         27.536102         99130.           684.3          34.2
         28.036102        100930.           684.1          34.1
         28.536102        102730.           683.9          34.1
         29.036102        104530.           683.8          34.1
         29.536102        106330.           683.6          34.1
         30.036102        108130.           683.4          34.1
GRAPH TITLED
    HERING,F.S., BOX 21, STRESS RELAXATION OF NYLON
    FRACTION OF INITIAL STRESS VS. TIME,HOURS
HAS BEEN PLOTTED.

        LAST =  0
```

APPENDIX E

PREDICTION OF STRESS-STRAIN CURVES USING THE EMPIRICAL CREEP EQUATION

A simple method for predicting creep curves from stress-strain curves is outlined in Figure 34. As indicated in Figure 34(a), tensile tests are run at a series of constant strain rates. A cross-plot, Figure 34(b), is constructed for reciprocal strain rate vs. strain at a constant stress. By the graphical integration shown in Figure 34(b), times corresponding to certain strains may be found for a given stress level, and creep curves plotted as illustrated in Figure 34(c). This method implies the existence of an equation of state relationship between strain, stress, and strain rate, but does not explicitly state such an equation. A similar scheme was proposed by Marin and Pao [30], who obtained good agreement from test data on polymethylmethacrylate.

In this study, the empirical creep equation was used to predict stress-strain curves by reversing the procedure outlined in Figure 34. From the empirical creep equation,

$$\epsilon = K\sigma^N + B\sigma^R \cdot t + P\sigma^M\left(1 - e^{-ct^Q}\right) \tag{E.1}$$

The slope (strain rate) at any time for a given stress may be found by:

$$\frac{d\epsilon}{dt} = B\sigma^R + P\sigma^M cQt^{Q-1} e^{-ct^Q} \tag{E.2}$$

For a specified strain rate and a given stress, equation (E.2) may be solved implicitly for time, and this result substituted in equation (E.1) to obtain the corresponding strain. A Fortran IV computer program, called SIGEPS, was used to perform these calculations. Output from SIGEPS consisted of stress-strain data for a material for a given strain rate.

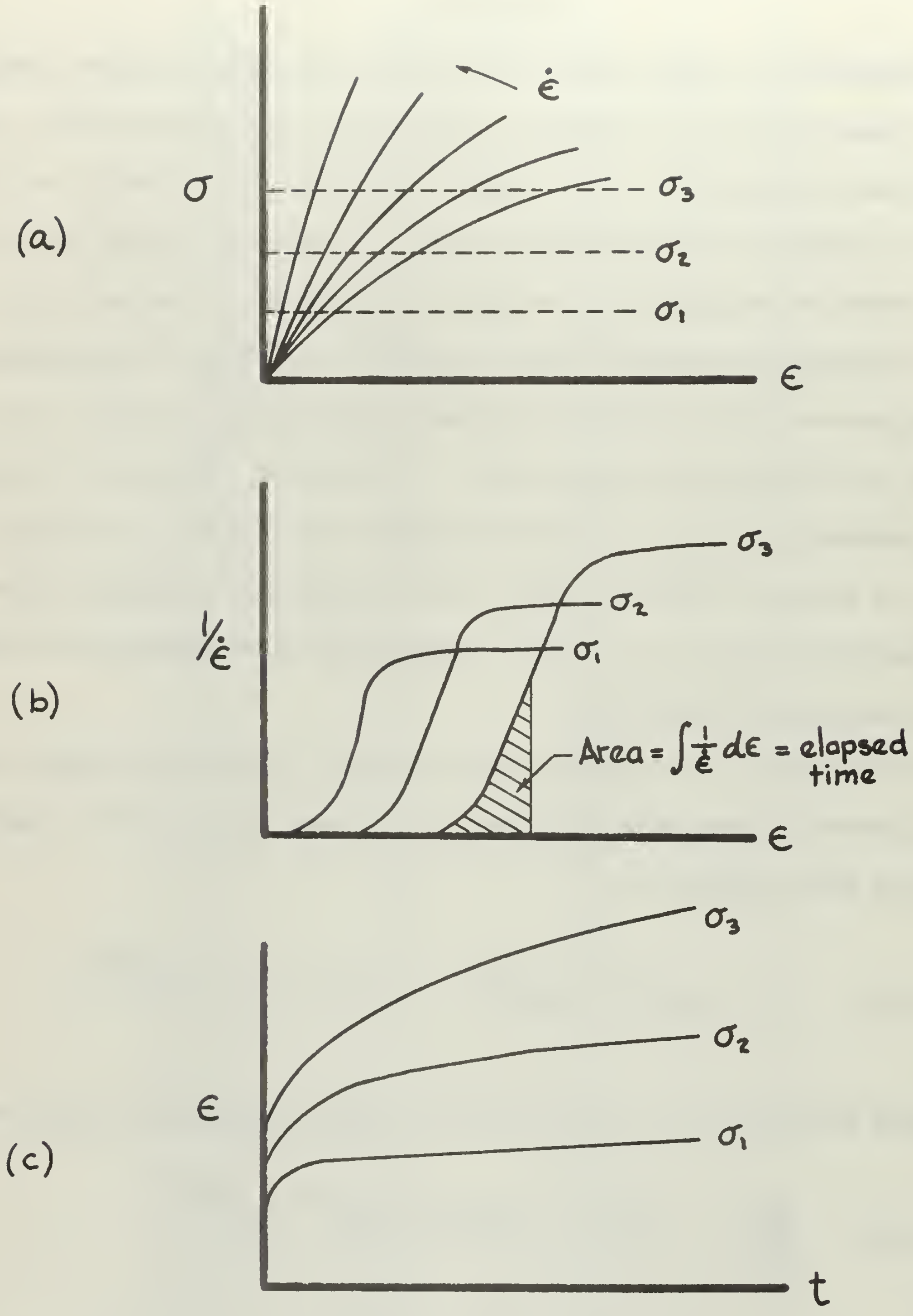


Figure 34

Prediction of Creep Curves from Stress-Strain Data

Program SIGEPS and typical output is given on the following pages. Figure 35 displays this output along with experimental tensile test data obtained at the same strain rate. The same characteristic was observed for most materials tested; the predicted tensile strain is greater than the observed strain for a given stress level. The deviation from the predicted curve becomes greater at higher stress levels. In the tensile test, stress changes with time, and internal structure changes probably develop. These changes are not accounted for in the creep equation used for the prediction. In addition, it should be noted that the tensile tests took place over a time span of about 10 minutes whereas the time duration of the creep test was 100 hours. Consequently the prediction of stress-strain curves is based on the creep curve at very short times. As indicated in the tabulated output from program FITDATA (See Appendix B), it is at these short times where poorest fit of equation (E.1) usually occurred.

Figure 36 illustrates an exception to the previous discussion. The predicted stress-strain curves for polycarbonate showed excellent agreement with tensile test data. It is possible that structure factor effects in polycarbonate are negligible at the stress levels encountered. Furthermore, output from FITDATA for polycarbonate showed good agreement between the empirical creep equation and the test data at times under 10 minutes.

It would be useful to further examine the feasibility of this method by utilizing an empirical creep equation specifically designed to give good data fits at short times.

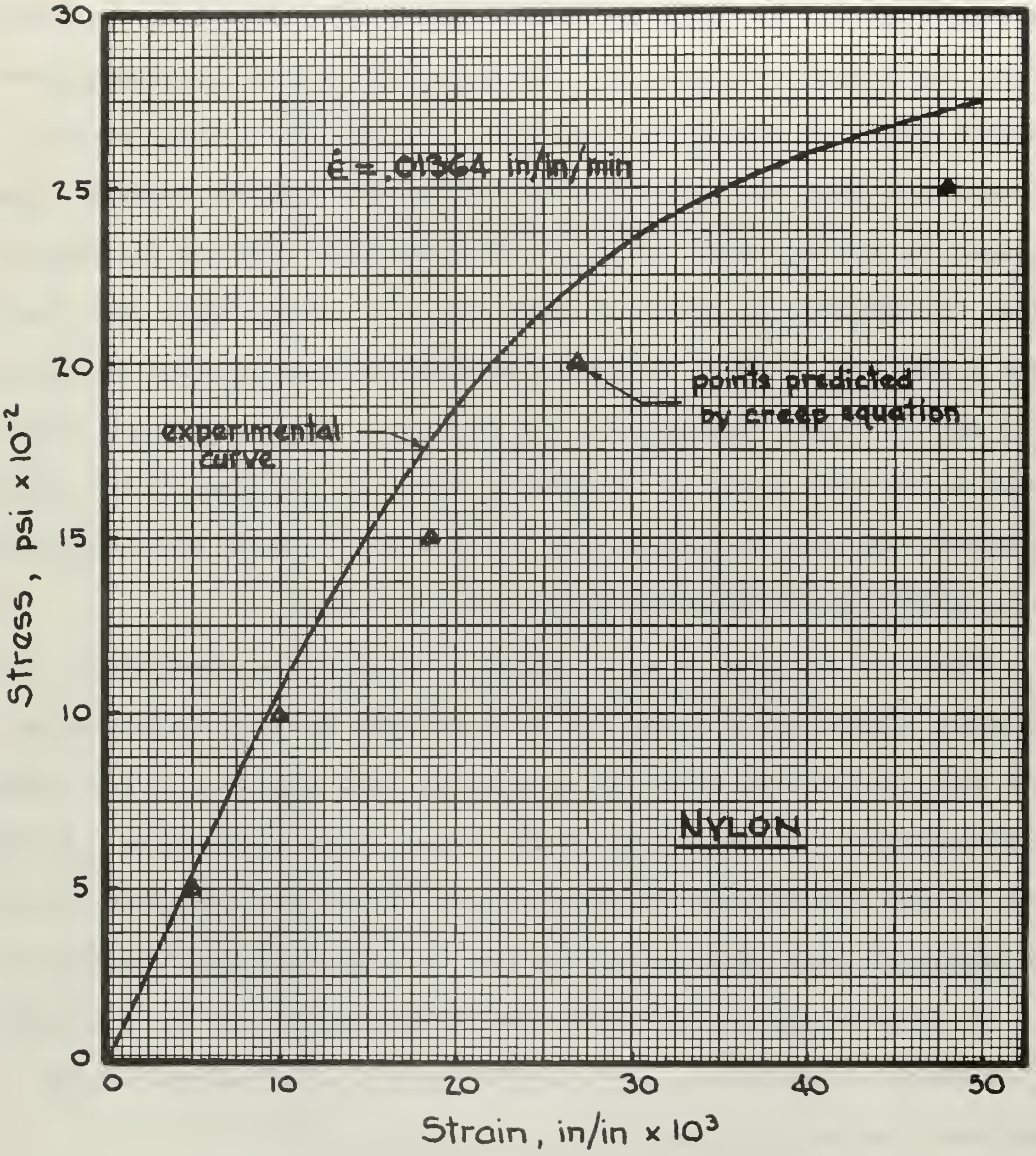


Figure 35

Predicted Stress-Strain Curve for Nylon

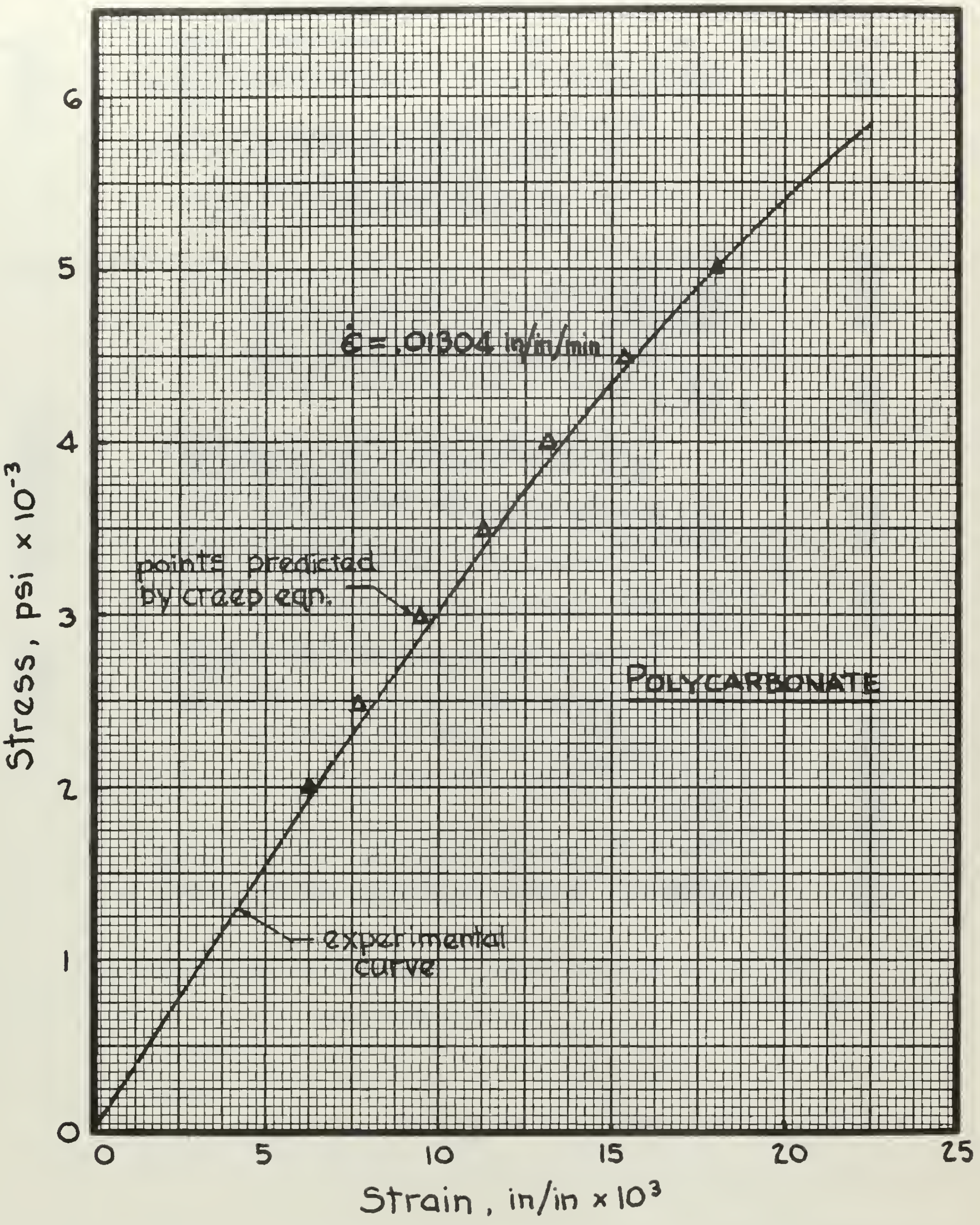
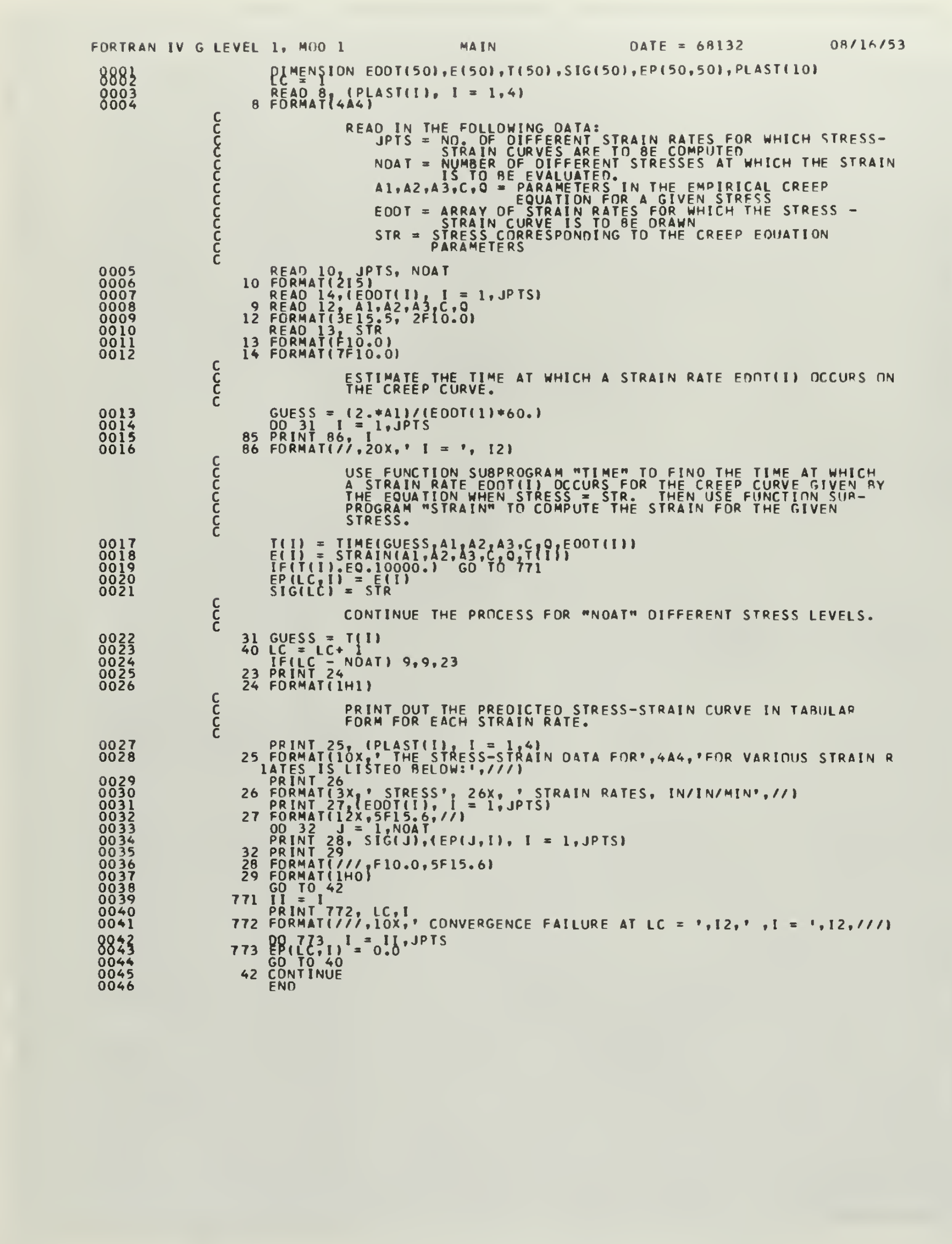


Figure 36

Predicted Stress-Strain Curve for Polycarbonate

COMPUTER PROGRAM "SIGEPS"



```
FORTRAN IV G LEVEL 1, MOD 1              MAIN                DATE = 68132          08/16/53                    PAGE 0001

           DIMENSION EDOT(50),E(50),T(50),SIG(50),EP(50,50),PLAST(10)
           LC = 1
           READ 8, (PLAST(I), I = 1,4)
         8 FORMAT(4A4)
C
C                  READ IN THE FOLLOWING DATA:
C                     JPTS = NO. OF DIFFERENT STRAIN RATES FOR WHICH STRESS-
C                            STRAIN CURVES ARE TO 8E COMPUTED
C                     NOAT = NUMBER OF DIFFERENT STRESSES AT WHICH THE STRAIN
C                            IS TO 8E EVALUATED.
C                     A1,A2,A3,C,Q = PARAMETERS IN THE EMPIRICAL CREEP
C                                    EQUATION FOR A GIVEN STRESS
C                     EDOT = ARRAY OF STRAIN RATES FOR WHICH THE STRESS -
C                            STRAIN CURVE IS TO 8E DRAWN
C                     STR = STRESS CORRESPONDING TO THE CREEP EQUATION
C                           PARAMETERS
C
           READ 10, JPTS, NOAT
        10 FORMAT(2I5)
           READ 14,(EDOT(I), I = 1,JPTS)
         9 READ 12, A1,A2,A3,C,Q
        12 FORMAT(3E15.5, 2F10.0)
           READ 13, STR
        13 FORMAT(F10.0)
        14 FORMAT(7F10.0)
C
C                  ESTIMATE THE TIME AT WHICH A STRAIN RATE EDOT(I) OCCURS ON
C                  THE CREEP CURVE.
C
           GUESS = (2.*A1)/(EDOT(1)*60.)
           OO 31  I = 1,JPTS
        85 PRINT 86, I
        86 FORMAT(//,20X,' I = ', I2)
C
C                  USE FUNCTION SU8PROGRAM "TIME" TO FINO THE TIME AT WHICH
C                  A STRAIN RATE EDOT(I) OCCURS FOR THE CREEP CURVE GIVEN BY
C                  THE EQUATION WHEN STRESS = STR.  THEN USE FUNCTION SUB-
C                  PROGRAM "STRAIN" TO COMPUTE THE STRAIN FOR THE GIVEN
C                  STRESS.
C
           T(I) = TIME(GUESS,A1,A2,A3,C,Q,EDOT(I))
           E(I) = STRAIN(A1,A2,A3,C,Q,T(I))
           IF(T(I).EQ.10000.)  GO TO 771
           EP(LC,I) = E(I)
           SIG(LC) = STR
C
C                  CONTINUE THE PROCESS FOR "NOAT" DIFFERENT STRESS LEVELS.
C
        31 GUESS = T(I)
        40 LC = LC+ 1
           IF(LC - NOAT) 9,9,23
        23 PRINT 24
        24 FORMAT(1H1)
C
C                  PRINT OUT THE PREDICTED STRESS-STRAIN CURVE IN TABULAR
C                  FORM FOR EACH STRAIN RATE.
C
           PRINT 25, (PLAST(I), I = 1,4)
        25 FORMAT(10X,' THE STRESS-STRAIN DATA FOR',4A4,'FOR VARIOUS STRAIN R
          1ATES IS LISTEO 8ELOW:',///)
           PRINT 26
        26 FORMAT(3X,' STRESS', 26X, ' STRAIN RATES, IN/IN/MIN',//)
           PRINT 27,(EDOT(I), I = 1,JPTS)
        27 FORMAT(12X,5F15.6,//)
           OO 32  J = 1,NOAT
           PRINT 28, SIG(J),(EP(J,I), I = 1,JPTS)
        32 PRINT 29
        28 FORMAT(///,F10.0,5F15.6)
        29 FORMAT(1H0)
           GO TO 42
       771 II = I
           PRINT 772, LC,I
       772 FORMAT(///,10X,' CONVERGENCE FAILURE AT LC = ',I2,' ,I = ',I2,///)
           DO 773  I = II,JPTS
       773 EP(LC,I) = 0.0
           GO TO 40
        42 CONTINUE
           END
```

```
FORTRAN IV G LEVEL 1, MOD 1           STRAIN          DATE = 68132        08/16/53              PAGE 0001

                  FUNCTION STRAIN(A1,A2,A3,C,Q,Z)
            C
            C             FOR A GIVEN TIME (Z) THIS FUNCTION SUBPROGRAM USES THE
            C             EMPIRICAL CREEP EQUATION FOR A GIVEN STRESS TO COMPUTE
            C             THE CORRESPONDING VALUE OF STRAIN.
            C
                  POWER = C*(Z**Q)
                  IF(POWER - 50.) 62,62,61
               61 PRODT = 1.0
                  GO TO 63
               62 PRODT = 1.-EXP(-POWER)
               63 STRAIN = A1 + A2*Z+A3*PRODT
                  RETURN
                  END
```

```
FORTRAN IV G LEVEL 1, MOD 1           TIME            DATE = 68132        08/16/53              PAGE 0001

                  FUNCTION TIME(X,A1,A2,A3,C,Q,EO)
            C
            C             THIS FUNCTION SUBPROGRAM COMPUTES THE TIME (X) AT WHICH A
            C             GIVEN STRAIN RATE (EO) OCCURS, BASED ON THE CREEP PARAM-
            C             ETERS A1,A2,A3,C,Q FOR A GIVEN STRESS.  THE FUNCTION
            C             CALLED "FUNCT" IS THE DERIVATIVE OF THE CREEP EQUATION,
            C             DE/DT, MINUS THE GIVEN STRAIN RATE,ED.  THE SUBPROGRAM
            C             USES A NEWTON-RAPHSON ITERATION TO FIND THE UNKNOWN TIME
            C             (X), WHICH IS THE ROOT OF THE EQUATION"FUNCT = 0".
            C             "DERIV" IS USED IN THE N/R ITERATION AND IS THE DERIVATIVE
            C             OF "FUNCT" TAKEN ANALYTICALLY.
            C
                  L = 0
                  R = Q - 1.
               90 XOLD = X
                  PRINT 87,L,XOLD
               87 FORMAT(25X,' L = ',I2,'    XOLD = ',E15.9)
                  PWR = C*(X**Q)
                  IF(PWR - 50.) 80,80,82
               80 PROD = A3*EXP(-PWR)
                  FUNCT = A2 - EO + (C*Q*PROD*(X**R))
                  DERIV = PROD*C*Q*(R*(X**(R-1.)) - C*Q*(X**(2.*R)))
                  X = X - (FUNCT/DERIV)
                  IF(X.LT.0.0)  X =XOLD/2.
                  DIFF = ABS(X - XOLD)
                  PRINT 93,DIFF,FUNCT,DERIV,PWR
               93 FORMAT(25X,' DIFF = ',F15.9,'    FUNCT = ',E15.9,' DERIV = ',E15.9
                 1,'    PWR = ',F10.6)
                  L = L+1
            C
            C             IF MORE THAN 20 ITERATIONS ARE USED TO FIND THE
            C             TIME (X), ASSUME CONVERGENCE FAILURE.
            C
                  IF(L.GT.20)  GO TO 82
                  IF(DIFF - .00001) 81,81,90
               81 TIME = X
                  RETURN
               82 TIME = 10000.
                  RETURN
                  END
```

TABULATED OUTPUT FROM "SIGEPS"

```
        THE STRESS-STRAIN DATA FOR  POLYCARBONATE FOR VARIOUS STRAIN RATES IS LISTED BELOW:

 STRESS                           STRAIN RATES, IN/IN/MIN

              0.013040


  2000.     0.006087


  2500.     0.007790


  3000.     0.009535


  3500.     0.011291


  4000.     0.013212


  4500.     0.015322


  5000.     0.018104
```

APPENDIX F

THE BOX DISTRIBUTION FUNCTION FOR RETARDATION SPECTRA

The use of a continuous distribution of retardation times to characterize delayed elastic response was discussed in Section 6. Not only do theoretical considerations favor this approach, but mathematical solutions to viscoelastic problems may be considerably simplified, particularly if the functions representing the spectra can be handled by transform techniques. As an example, an analytic expression for creep behavior will be derived here, using a box distribution of retardation times as illustrated in Figure 37.

An expression for creep strain corresponding to the four-element model of Figure 10, page 25 , may be written:

$$\epsilon = A_1 + A_2 t + A_3 (1 - e^{-bt}) \tag{F.1}$$

where the constant b is the rate constant equal to the reciprocal of the characteristic relaxation time τ , and coefficients A_1, A_2, and A_3, depend on the material and the applied stress. For a material with many discreet retardation times, the expression for creep strain given by equation (F.1) becomes:

$$\epsilon = A_1 + A_2 t + A_3 \sum_i a_i (1 - e^{-b_i t}) \tag{F.2}$$

where the constant b_i is the rate constant associated with the $i^{\underline{th}}$ retardation time τ_i , and the factors a_i are weighting constants indicating the relative contribution of the $i^{\underline{th}}$ mechanism to the delayed strain. Since the factors a_i are chosen so that $\sum a_i = 1$, equation (F.2) may be rearranged as:

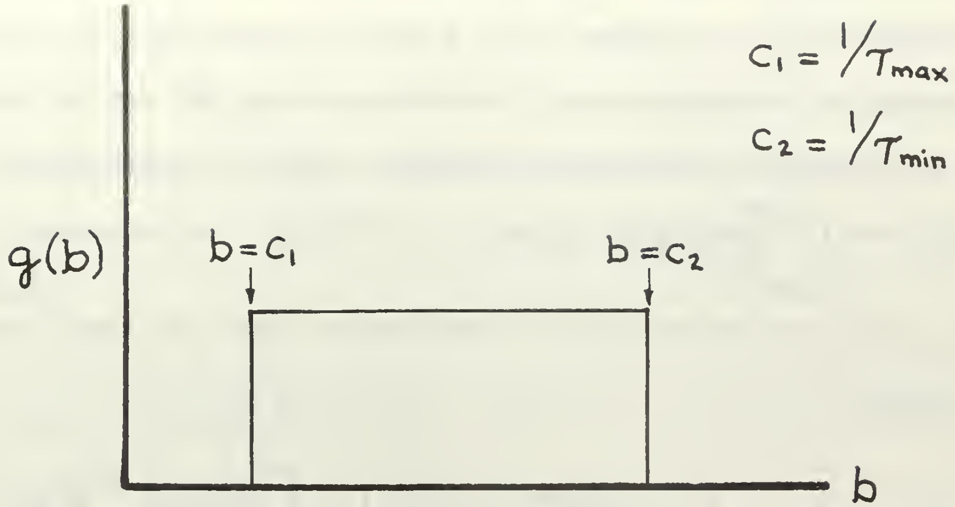


Figure 37

Box Distribution of Rate Constants

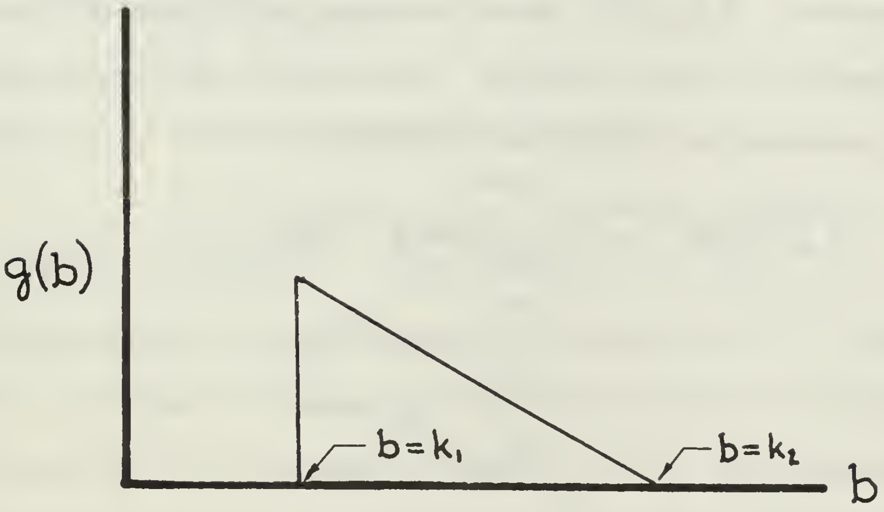


Figure 38

Wedge Distribution of Rate Constants

$$\text{(F.3)} \quad \epsilon = A_1 + A_2 t + A_3 \overbrace{\left(1 - \sum_i a_i e^{-b_i t}\right)}^{\epsilon_d}$$

If the delayed elastic response (ϵ_d) has associated with it a continuous spectrum of retardation times, then the factors a_i may be represented by a rate constant distribution function, $g(b)$, such that:

$$\text{(F.4)} \quad \int_{b=0}^{b=\infty} g(b)\, db = 1$$

For this continuous distribution of retardation times the creep equation (F.3) becomes:

$$\text{(F.5)} \quad \epsilon = A_1 + A_2 t + A_3\left(1 - \int_0^\infty g(b)\, e^{-bt}\, db\right)$$

Equation (F.5) now contains the Laplace transformation of the rate constant distribution function $g(b)$, and may be rewritten as:

$$\text{(F.6)} \quad \epsilon = A_1 + A_2 t + A_3\left(1 - \mathcal{L}\{g(b)\}\right)$$

where $\mathcal{L}\{g(b)\}$ represents the Laplace transform of the function.

The transform $\mathcal{L}\{g(b)\}$ can be evaluated readily in the case of a box distribution of rate constants. Referring to the notation used for the box distribution illustrated in Figure 37,

$$\begin{aligned} \mathcal{L}\{g(b)\} &= \int_{b=0}^{b=\infty} g(b)\, e^{-bt}\, db \\ &= \int_{b=C_1}^{b=C_2} 1 \cdot e^{-bt}\, db \\ &= -\tfrac{1}{t}\, e^{-bt} \Big|_{b=C_1}^{b=C_2} \end{aligned}$$

$$\text{(F.7)} \quad \mathcal{L}\{g(b)\} = \tfrac{1}{t}\left(e^{-C_1 t} - e^{-C_2 t}\right)$$

In equation (F.7), constants C_2 and C_1 are the maximum and minimum rate constants, respectively, of the box distribution. These constants may be evaluated for a given set of creep data as shown in Figure 19.

In equations (F.1) and (F.6), the coefficient A_3 is intended to represent the delayed strain after elastic equilibrium is achieved, i.e., after infinite time. The delayed strain at time zero is zero. Therefore, the expression for $\mathcal{L}\{g(b)\}$ given by equation (F.7) must be normalized so that:

(F.8) $$\mathcal{L}\{g(b)\} = 0 \quad , \quad t = 0$$

(F.9) $$\mathcal{L}\{g(b)\} = 1 \quad , \quad t = \infty$$

This may be accomplished by expanding the exponential terms of equation (F.7) in a MacLaurin series:

$$\mathcal{L}\{g(b)\} = \frac{1}{t}\left[1 - c_1 t + \frac{(c_1 t)^2}{2!} - \frac{(c_1 t)^3}{3!} \cdots - \left(1 - c_2 t + \frac{(c_2 t)^2}{2!} - \frac{(c_2 t)^3}{3!} + \cdots\right)\right]$$

$$= \left[c_2 - c_1 + \frac{c_1^2 t}{2!} - \frac{c_2^2 t}{3!} + \cdots\right]$$

(F.10) $$\lim_{t \to 0} \mathcal{L}\{g(b)\} = c_2 - c_1$$

Equation (F.7) may now be normalized to get:

(F.11) $$\mathcal{L}_N\{g(b)\} = \frac{1}{(c_2 - c_1) \cdot t}\left[e^{-c_1 t} - e^{-c_2 t}\right]$$

By substitution of the time limits $t = 0$ and $t = \infty$ into equation (F.11) it is apparent that the conditions of equations (F.8) and (F.9) are satisfied. Substitution of equation (F.11) into equation (F.6) gives the analytic expression for creep strain when the retardation spectrum is a box distribution of rate constants.

(F.12) $$\epsilon = A_1 + A_2 t + A_3\left[1 - \frac{e^{-c_1 t} - e^{-c_2 t}}{(c_2 - c_1) \cdot t}\right]$$

Similar derivations may be performed whenever the retardation spectrum can be represented by a sum of functions whose transforms can be found. Unfortunately, the resulting expression is quite complex for all but the simplest distribution functions. For example, the wedge distribution function, Figure 38, similar to the theoretical model of Alfrey discussed in Section 6, leads to the expression:

$$\text{(F.13)} \quad \epsilon = A_1 + A_2 t + A_3\left[1 - \frac{e^{-k_1 t}(k_2 t - k_1 t - 1) + e^{-k_2 t}}{\frac{1}{2} t^2 (k_1^2 - k_2^2 - k_1 k_2)}\right]$$

In the equations for creep strain developed in this Appendix, the delayed elastic response, ϵ_d , represents a portion of the total strain. In equation (F.1), for example,

$$\text{(F.14)} \quad \epsilon_d = A_3(1 - e^{-bt})$$

The value of the strain ϵ_d varies from zero at $t = 0$ to a value A_3 at $t = \infty$. It is often convenient to normalize the delayed strain. The normalized form, called the delayed strain fraction, is defined by:

$$\text{(F.15)} \quad \varepsilon = \epsilon_d / A_3$$

The delayed strain fraction varies from $\varepsilon = 0$ at $t = 0$ to $\varepsilon = 1$ at $t = \infty$, and its value at any time $t > 0$ represents the progress toward elastic equilibrium. For equation (F.1), the delayed strain fraction is:

$$\text{(F.16)} \quad \varepsilon = 1 - e^{-bt}$$

A better insight into the nature of the delayed elastic response is achieved by plotting delayed strain fraction versus the logarithm of time. Equation (F.16) is plotted in this manner in Figure 39. The inflection point of this sigmoidal shaped curve is reached at a time $t = 1/b$, the characteristic retardation time. The delayed strain fraction at $t = 1/b$ is $\mathcal{E} = 1 - 1/e = 0.632$. The slope of this curve changes very gradually in the range $\mathcal{E} = 0.3$ to $\mathcal{E} = 0.8$. In this region, the shape of the curve can be approximated by a straight line drawn with a slope corresponding to $d\mathcal{E}/d(\ln t)$ evaluated at $\mathcal{E} = 0.5$. This is shown by line segment A-A in Figure 39.

For a box distribution of rate constants, the delayed strain fraction versus log time curve (Figure 40) is also sigmoidal in shape, with even less drastic slope changes in the region around $\mathcal{E} = 0.5$. In order to accurately depict the delayed elastic response, the sigmoidal curve must fit the data closely at a time t_1 corresponding to $\mathcal{E} = 0.5$. Otherwise, large errors in predicted strain occur over most of the time range. Figure 41(a) illustrates a sigmoidal curve exhibiting a slope mismatch with a data plot. In Figure 41(b), the slopes of the data plot and the sigmoidal function are the same, but the responses do not occur at the same time. It is apparent from these figures that the error in predicted delayed strain fraction will be large if there is a slope and/or time mismatch at approximately $\mathcal{E} = 0.5$.

Plots of $\mathcal{E}$ versus $\ln t$ were made for the creep data obtained in this investigation. For different viscoelastic materials tested, these plots exhibited remarkable similarities. The delayed strain fraction reached a value $\mathcal{E} = 0.5$ at a time $t \approx 1$ hour, and the slope of the curves at $\mathcal{E} = 0.5$ was $d\mathcal{E}/d(\ln t) \approx 0.15$. The same values were observed for PP, PC, NY, and PE and were independent of stress. Therefore, the same function may be used to characterize the delayed elastic response of each of these materials. A question arises as to whether or not a function such as the box distribution leads to a suitable representation. To text the applicability of the box distribution, it is necessary to investigate the characteristics of the resulting sigmoidal curve for the limiting

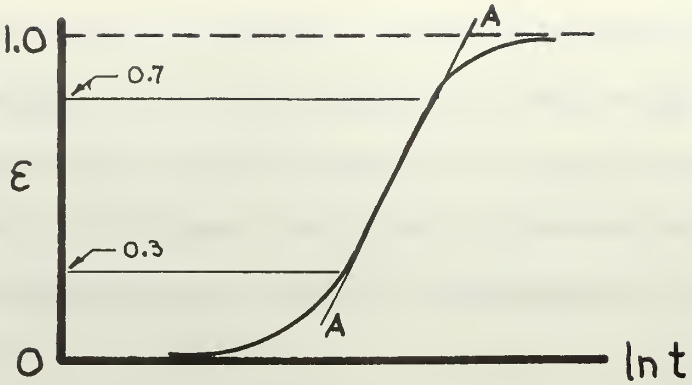


Figure 39

Delayed Strain Fraction vs. Ln Time for a Single Retardation Time

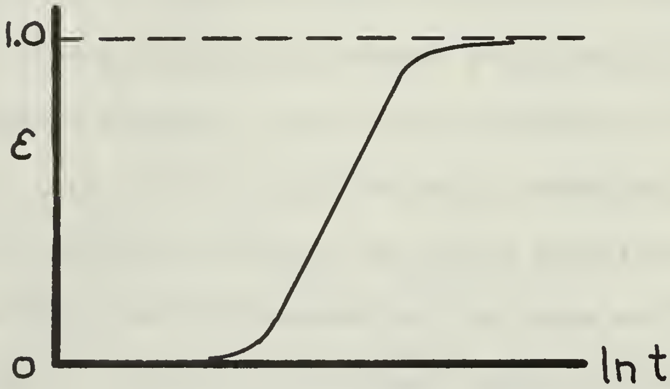


Figure 40

Sigmoidal Curve for a Box Distribution

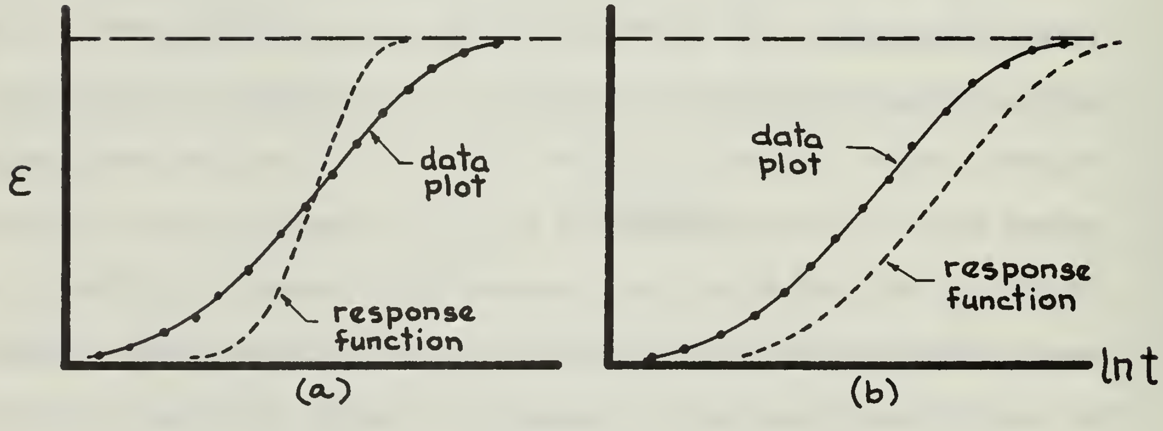


Figure 41

Unsuitable Representations of Delayed Elastic Response cases of the box distribution shown in Figure 42. This analysis is given below:

If the limits of the box distribution are extended to cover many decades of time, as shown schematically in Figure 42(a),

$$C_2 \gg C_1$$

$$e^{-C_2 t} \ll e^{-C_1 t} \qquad \text{for} \quad t > 0$$

The expression for the delayed strain fraction (from equation (F.12)) is given by:

$$\varepsilon = 1 - \left[\frac{e^{-C_1 t} - e^{-C_2 t}}{(C_2 - C_1) \cdot t} \right] \tag{F.17}$$

The terms $e^{-C_2 t}$ and C_1 may be neglected so that,

$$\varepsilon = 1 - \frac{e^{-C_1 t}}{C_2 t} \tag{F.18}$$

The constant C_1 is very small so that,

$$e^{-C_1 t} \approx 1$$

The delayed strain fraction for the extended box distribution may then be approximated by:

$$\varepsilon = 1 - \frac{1}{C_2 t} \quad , \quad C_2 \gg C_1 \tag{F.19}$$

This curve is illustrated in Figure 43(a). The approximation is obviously in error for times $t < 1/C_2$, but the constant C_2 is large and the equation (F.19) is valid in the time region of interest.

For the case shown in Figure 42(b) where the limits converge and the box becomes narrow, the equation for delayed strain may be approximated as follows: Denoting the bracketed term of equation (F.17) by Z, and expanding the exponentials,

$$Z = \frac{1 - \frac{(C_1 t)^1}{1!} + \frac{(C_1 t)^2}{2!} - \frac{(C_1 t)^3}{3!} \cdots - 1 + \frac{(C_2 t)^1}{1!} - \frac{(C_2 t)^2}{2!} + \frac{(C_2 t)^3}{3!} \cdots}{(C_2 - C_1) \cdot t}$$

(F.20) $$Z = \frac{(C_2 - C_1)}{1!(C_2 - C_1)} - \frac{(C_2^2 - C_1^2)}{2!(C_2 - C_1)} \cdot t + \frac{(C_2^3 - C_1^3)}{3!(C_2 - C_1)} \cdot t^2 \ldots$$

The n^{th} term of this expansion is represented by:

$$\frac{C_2^n - C_1^n}{n!(C_2 - C_1)} \cdot t^{n-1}$$

The value of this term for the case of converging limits ($C_2 \to C_1$) may be determined by considering C_2 and C_1 to differ by a small quantity ϵ.

$$C_2 \to C_1 \quad . \quad C_2 = C_1 + \epsilon$$

then

$$C_2^n = (C_1 + \epsilon)^n$$

Using a binomial expansion and truncating after the second term,

$$C_1^n = (C_1 + \epsilon)^n = C_1^n + n\epsilon C_1^{n-1} ; \quad \text{for } \epsilon \ll C_1$$

then, $$\frac{C_2^n - C_1^n}{n!(C_2 - C_1)} = \frac{C_1^n + n\epsilon C_1^{n-1} - C_1^n}{n!(C_1 + \epsilon - C_1)} = \frac{n\epsilon C_1^{n-1}}{n!\epsilon} = \frac{C_1^{n-1}}{(n-1)!}$$

therefore,

(F.21) $$\lim_{C_2 \to C_1} \frac{C_2^n - C_1^n}{n!(C_2 - C_1)} = \frac{C^{n-1}}{(n-1)!}$$

Substituting equation (F.21) for the n^{th} term of equation (F.20) and evaluating the terms for n = 1,2,3..... .the expansion becomes:

(F.22) $$Z = 1 - \frac{C_1 t}{1!} + \frac{C_1^2 t^2}{2!} \ldots\ldots \frac{C_1^{n-1}}{(n-1)!} t^{n-1}$$

Equation (F.22) represents the expansion of $e^{-C_1 t}$ in a MacLaurin series, so the desired approximation has been found:

(F.23) $$\mathcal{E} = 1 - e^{-C_1 t}$$

or $\mathcal{E} = 1 - e^{-C_2 t}$ for $C_2 \to C_1$

Equation (F.23) would result from a "single spike" distribution, corresponding to a single characteristic retardation time. The analysis verifies the expected result that equation (F.17) should reduce to equation (F.23) as the width of the box distribution becomes vanishingly small. This approximation is sketched in Figure 43(b).

In order to be used for an accurate representation of the data in this investigation, the slope of the sigmoidal plot of the box distribution must take on the value:

$$\text{(F.24)} \quad \left.\frac{d\varepsilon}{d(\ln t)}\right|_{\substack{\varepsilon=0.5 \\ t=1}} = 0.15$$

By the approximation for the limiting case of $C_2 \ggg C_1$, given by equation (F.19),

$$\varepsilon = 1 - \frac{1}{C_2 t}$$

$$\text{for } \varepsilon = 0.5, \; t = 1$$

$$C_2 = 2$$

$$\frac{d\varepsilon}{dt} = \frac{1}{C_2 t^2} \; ; \quad \frac{d\varepsilon}{d(\ln t)} = \frac{d\varepsilon}{dt} \cdot t$$

$$\frac{d\varepsilon}{d(\ln t)} = \frac{1}{C_2 t}$$

$$\text{(F.25)} \quad \left.\frac{d\varepsilon}{d(\ln t)}\right|_{\substack{\varepsilon=0.5 \\ t=1}} = 0.5$$

For the case of narrowing box width, ($C_2 \longrightarrow C_1$), given by equation (F.23):

$$\varepsilon = 1 - e^{-C_2 t}$$

$$\text{for } \varepsilon = 0.5, \; t = 1$$

$$C_2 \approx 0.7$$

$$\frac{d\varepsilon}{dt} = C_2 e^{-C_2 t}$$

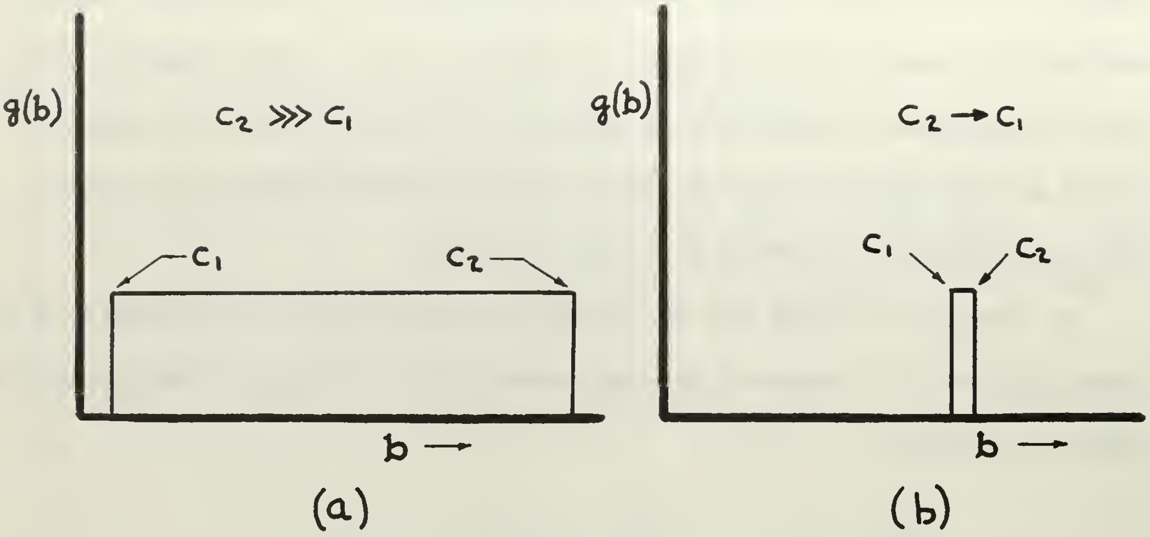


Figure 42

Limiting Cases of the Box Distribution

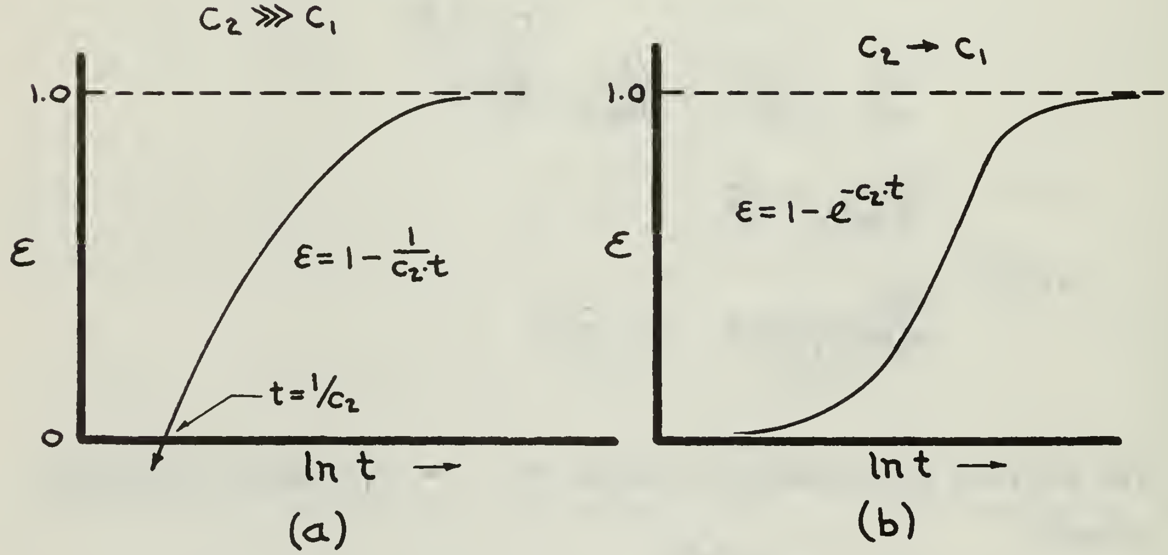


Figure 43

Approximations of the Response Function for the Limiting Cases of the Box Distribution

$$\frac{d\varepsilon}{d(\ln t)} = C_2 t e^{-C_2 t}$$

(F.26) $$\left.\frac{d\varepsilon}{d(\ln t)}\right|_{\substack{\varepsilon=0.5 \\ t=1}} \approx 0.35$$

It may be demonstrated by numerical methods that equation (F.26) represents the minimum value of the slope $d\varepsilon/d(\ln t)$ for any choice of the constants C_1 and C_2, provided $C_2 > C_1$. The requirements of equation (F.24) can not be met, since the observed slope of 0.15 does not fall in the range of 0.35 to 0.5. Therefore the box distribution is an unsatisfactory choice for the retardation spectrum of the materials tested in this experiment.

A similar analysis of the wedge distribution (Figure 38) shows a slope range $0.35 \leqslant \left(d\varepsilon/d(\ln t)\Big|_{\substack{\varepsilon=0.5 \\ t=1}}\right) \leqslant 0.4$ for all choices of K_1 and K_2, indicating that this rate constant distribution is also unsatisfactory. Since the limiting case of the box distribution given by equation (F.23) corresponds to the response of a single Voigt Element (Figure 5), the four parameter creep model (Equation (F.1)) gives a poor representation of delayed elastic strain.

As discussed in Section 6, good correlations can be obtained using a model with several discreet retardation times. This "multi-spike distribution" produces a plot of delayed strain fraction versus log time with inflections at each retardation time. These retardation times may be chosen so that the response function approximates the creep data quite closely. Figure 44 illustrates the response function resulting from a three-spike distribution superimposed on a typical plot of creep data. The response function from a box distribution is also shown. The lower limit, C_1, of the box is equal to the slow rate constant b_1 of the three-spike distribution, and the upper limit C_2 equals the fast rate constant b_3 .

These values for the box distribution function were chosen to illustrate the dominant role of the fast rate constants in governing the delayed elastic response. The constants C_1 and C_2 could be changed to shift the sigmoidal curve toward longer times, but the slope cannot be adjusted to produce a good fit with the data.

Monomodal distribution functions characterized by a single maximum can always be approximated by a sum of box-type distributions as shown in Figure 45. The limits of each succeeding box fall within the range of the first box. Since the slope of the delayed strain fraction versus log time plot resulting from such a distribution is restricted in accordance with equations (F.25) and (F.26), it is apparent that the retardation spectrum of the plastics tested is not monomodal.

Bimodal (or polymodal) spectra produce a series of inflections in the plot of ε vs. $\ln t$, which results in a lower overall slope, i.e., the delayed elastic response occurs over many decades of time. The three-spike distribution of Figure 44 is an example of a polymodal retardation spectrum.

The delayed elastic response of the materials tested covered about five decades of time. This response can be characterized by a polymodal retardation spectrum such as illustrated in Figure 46, where the peaks imply the presence of several different molecular mechanisms whose time constants differ by orders of magnitude. This model is in accord with Alfrey's concept (See Section 6) of local bond straightening and long-range uncoiling of the polymer chains.

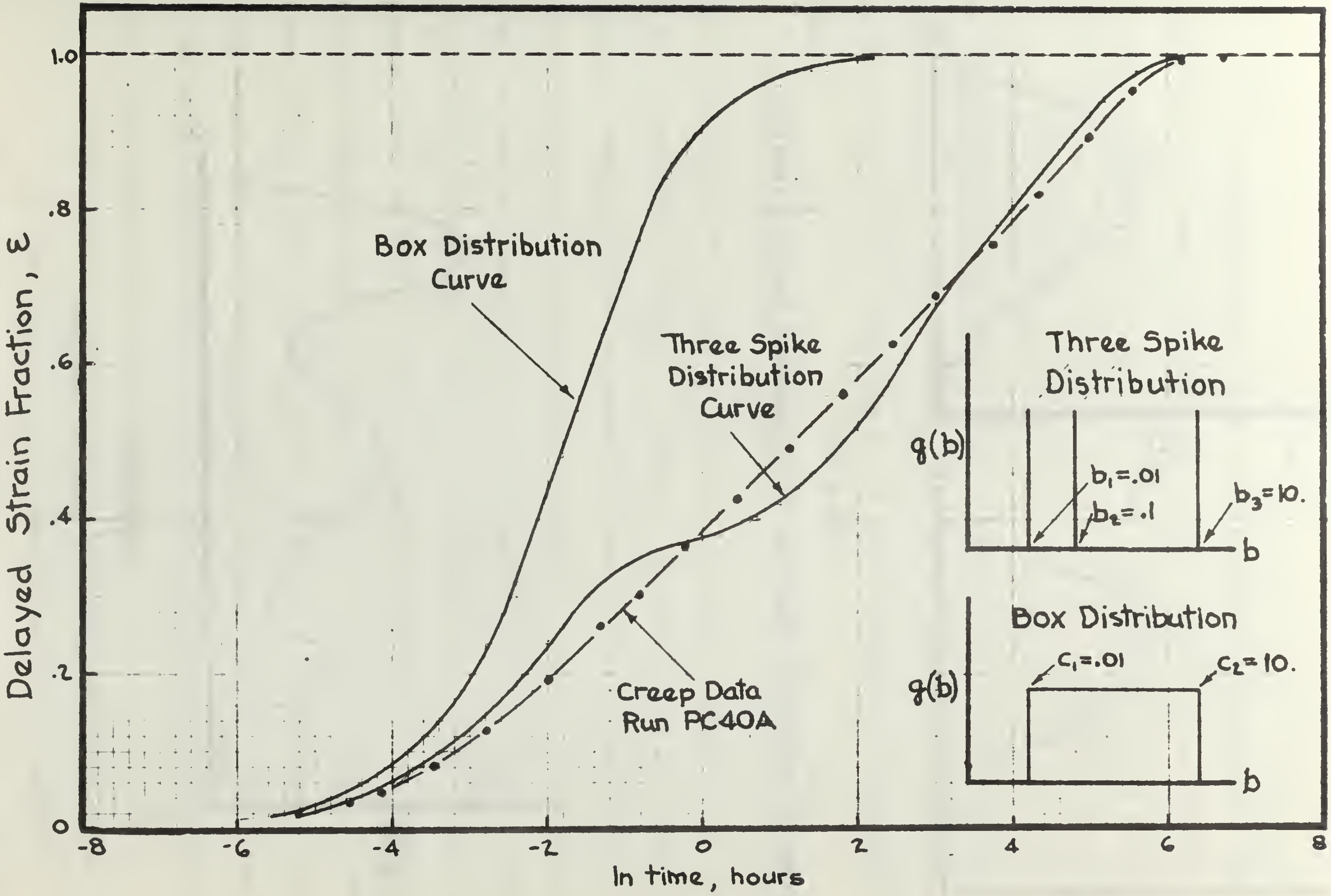


Figure 44

Delayed Elastic Response Functions Fitted to Creep Data

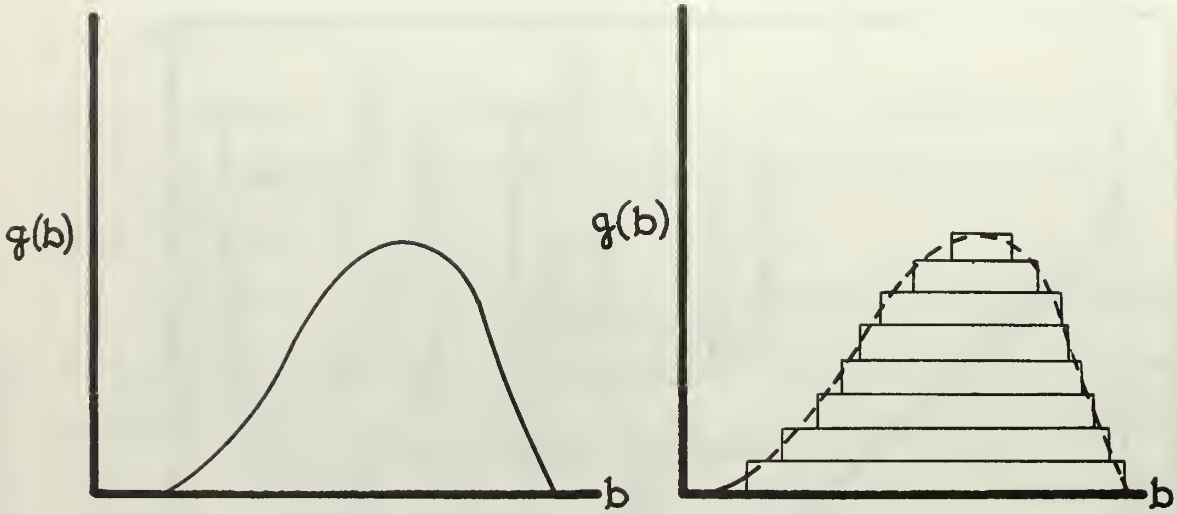


Figure 45

Monomodal Retardation Spectrum Approximated By Box Distributions

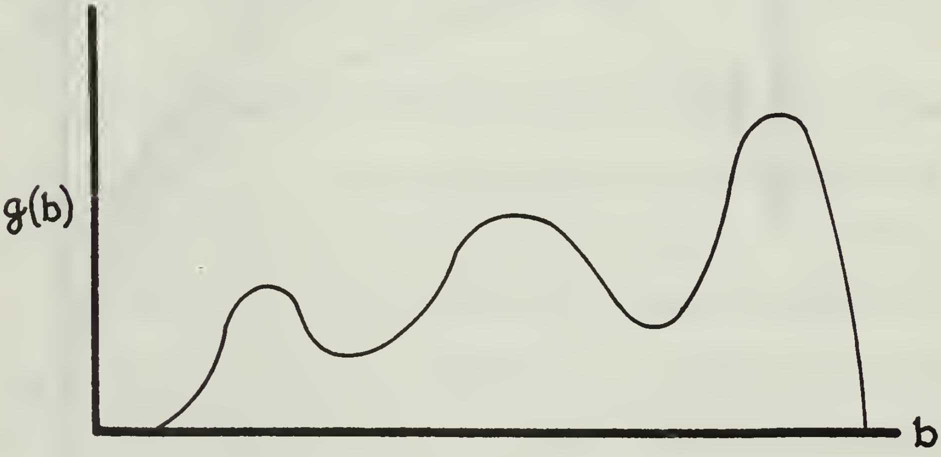


Figure 46

A Polymodal Distribution Function

APPENDIX G

EQUIPMENT

The creep machines used in this investigation apply constant dead-weight loading through calibrated lever arms. (Figure 47). Eleven identical units are installed in one test stand. Photographs of the laboratory and apparatus are shown in Figures 48 through 50. Specimens were gripped by steel pins placed through the 5/8 inch diameter hole in each end, and uniaxial loading was maintained through a series of universal joints above and below the grips. Turnbuckles and an adjustable counterbalance weight permitted final balancing of the lever arm prior to load application.

Studies of viscoelastic behavior of materials require large amounts of data obtained over long time periods of testing. Consequently, facilities for automatic recording of creep and relaxation data are highly desirable. Design of such instrumentation was started in conjunction with this project and is presently being installed. Special clip-on strain gage fixtures were fabricated to permit measurement of the large strains encountered in mechanical testing of viscoelastic materials. These clip gauges, shown in Figures 51 and 52 consist of phosphor bronze strips with BLH SR-4 Type A-5-1 strain gauges mounted on each side. Tests on phosphor bronze indicate that this material has remarkable linearity over the entire elastic range with no observable hysteresis effects. The strips are connected to aluminum legs which make a knife-edge contact with the specimen. This design has two useful features:

(1) The legs are detachable so that different leg lengths may be used depending on the strain sensitivity desired.

(2) The mode of attachment to the specimen is such that this electromechanical device may be used for specimens of various widths and thicknesses.

The four strain gauges are connected in a Wheatstone bridge arrangement shown in Figure 53. A potentiometer in the circuit permits a null adjustment to be made. This bridge circuitry has several advantages: [26]

(1) Automatic moment compensation (in the event there are deviations from axial loading.

(2) Signal strength increased by a factor of four. (over that of a single gauge mounted on the specimen)

Terminal boards have been set up on the eleven-unit creep machine to receive the leads from the extensometer circuit for each test specimen. Bridge circuits, each with its own balancing potentiometer, are placed in parallel across a D.C. power supply, with leads from each bridge entering master control panels. From these panels, the output signal from each extensometer may be fed into a multi-channel recorder, permitting testing and graphical data output for up to eleven specimens simultaneously. A stress analysis of the clip gauges, using linear elastic theory, is given in Figures 54 and 55.

It is desirable to have the output of electromechanical extensometers converted directly into digitalized form because of the usefulness of the digital computer in data analysis. Preliminary design of such instrumentation has begun at NPGS, Monterey.

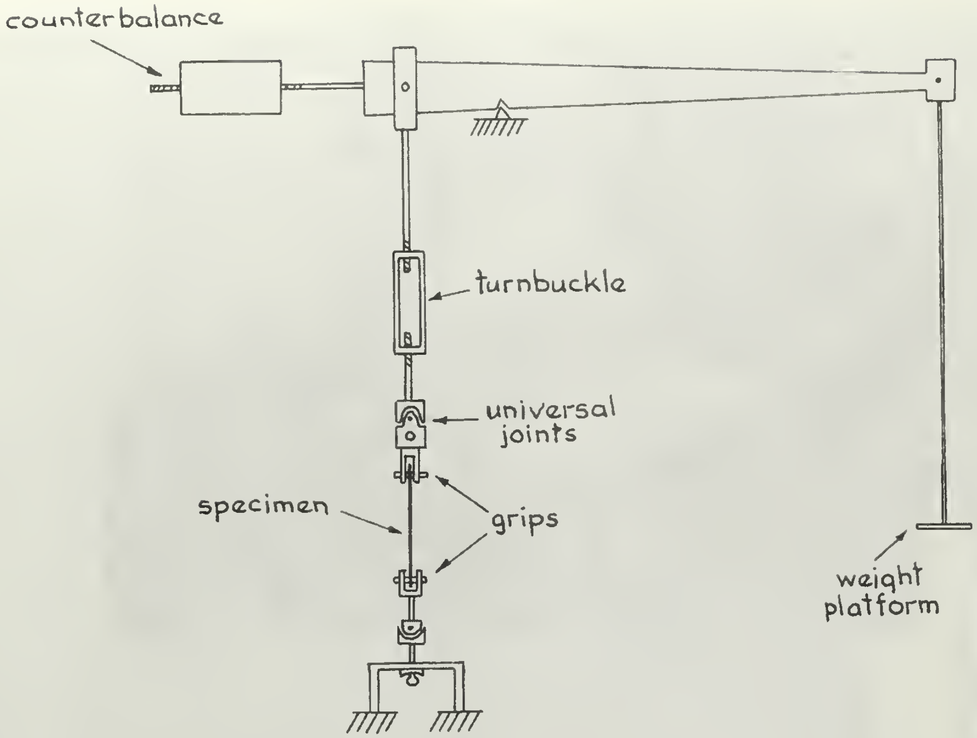


Figure 47

Lever Arm Loading Assembly

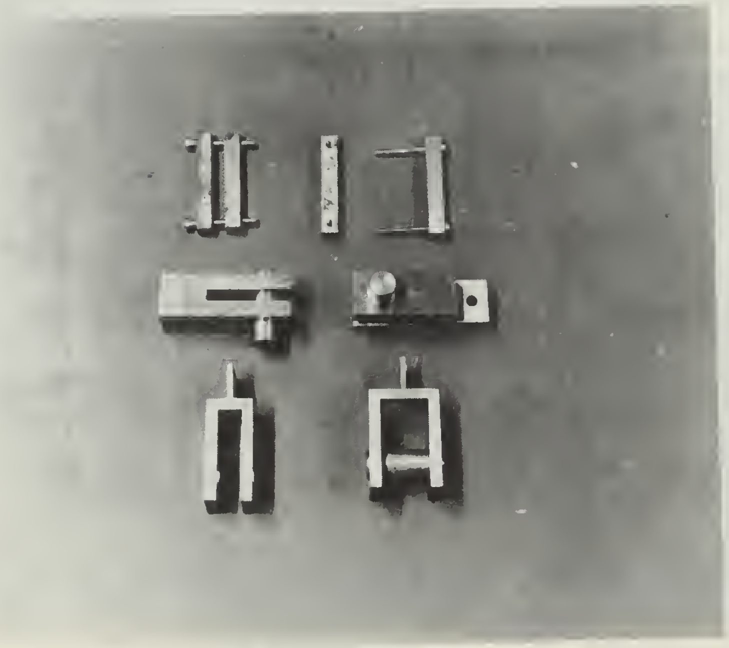


Figure 48

Specimen Grips and Clamps

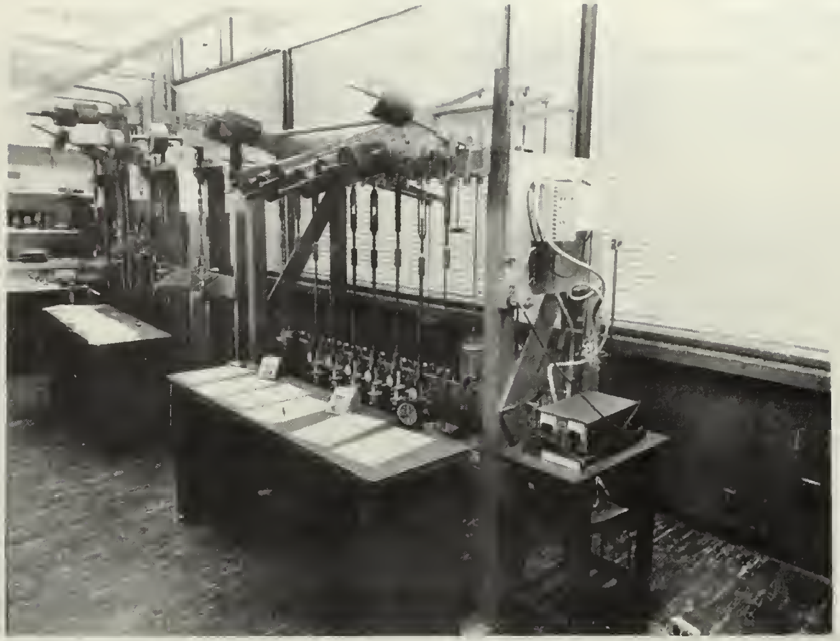


Figure 49

Eleven Unit Creep Testing Apparatus

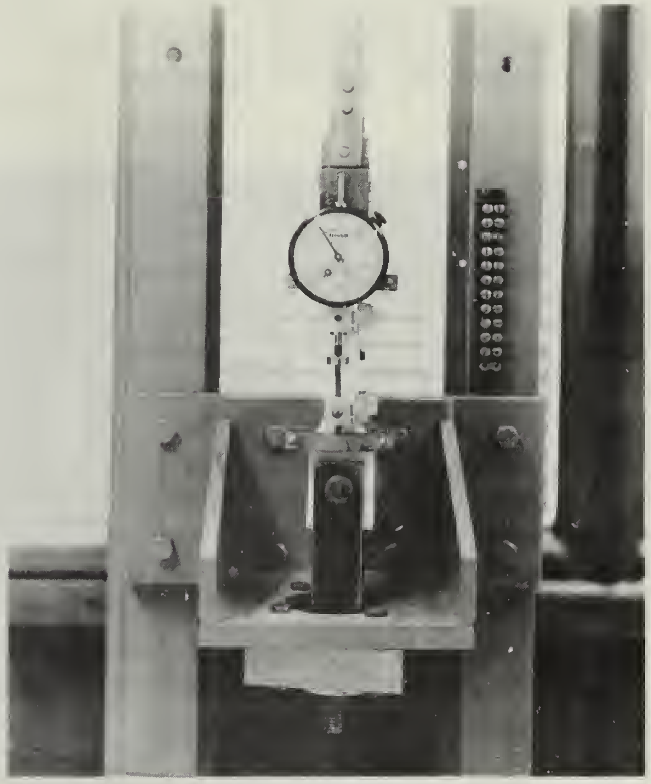


Figure 50

Tripolitis Extensometer Attached to Specimen

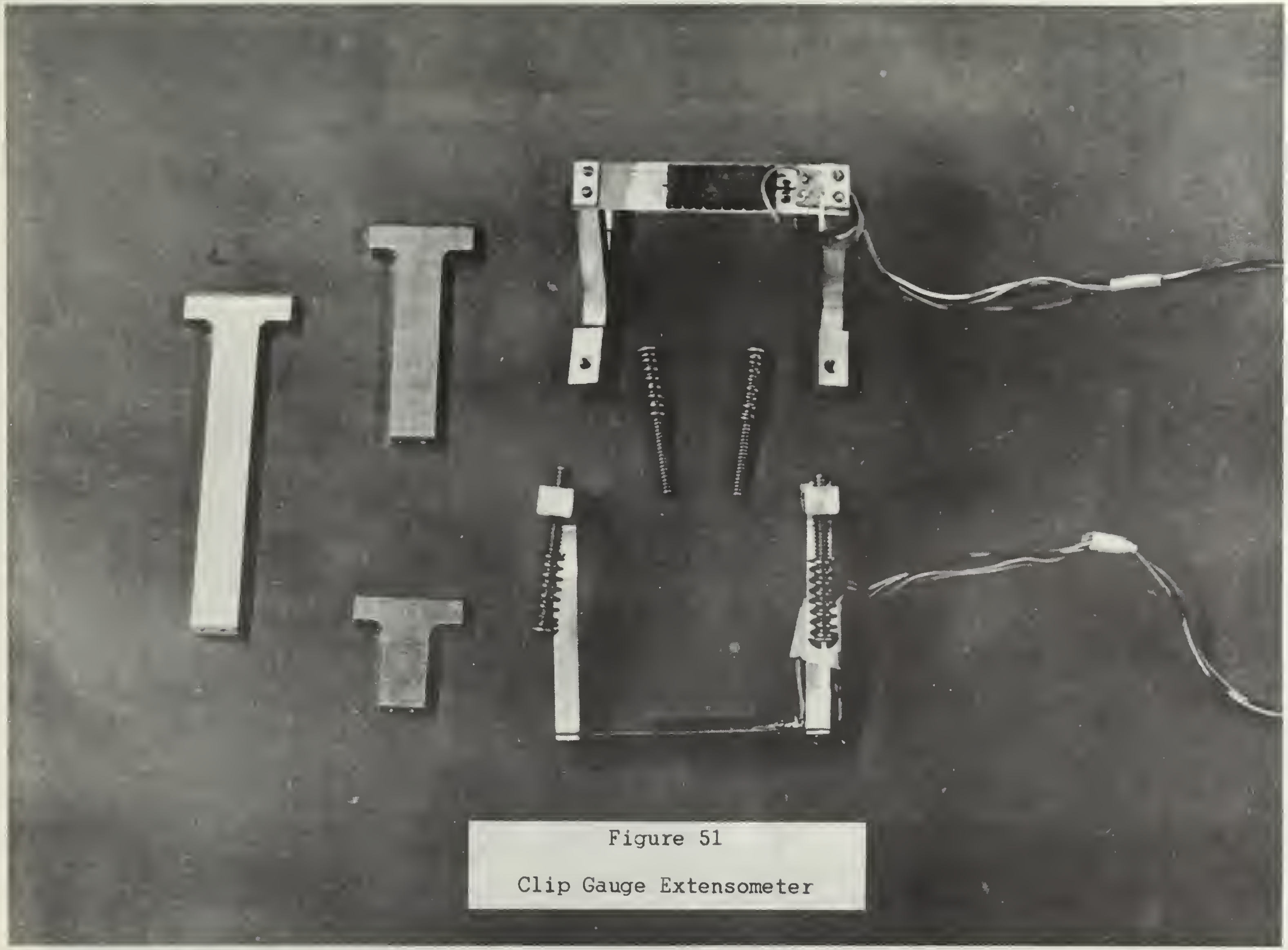


Figure 51

Clip Gauge Extensometer

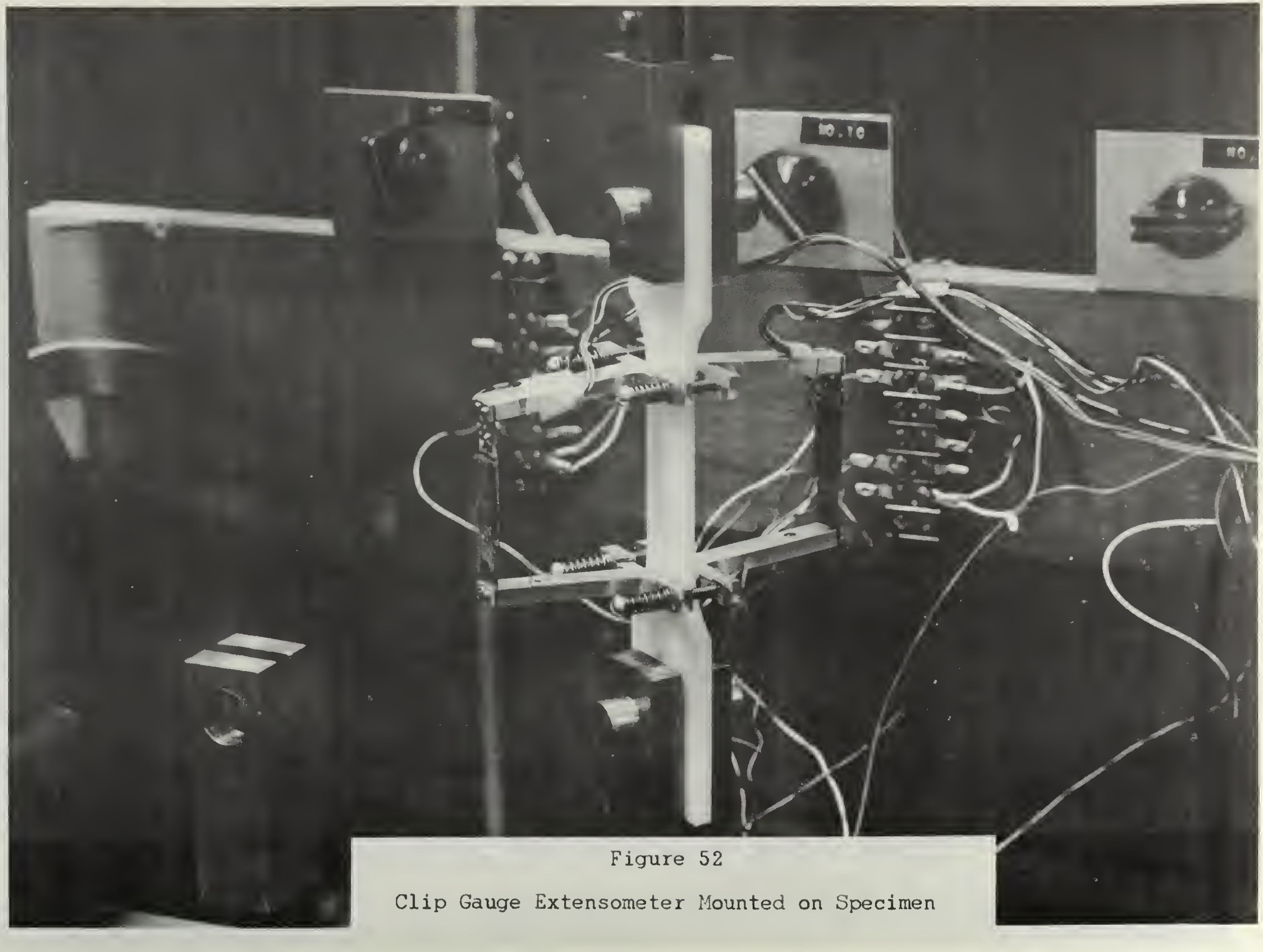


Figure 52

Clip Gauge Extensometer Mounted on Specimen

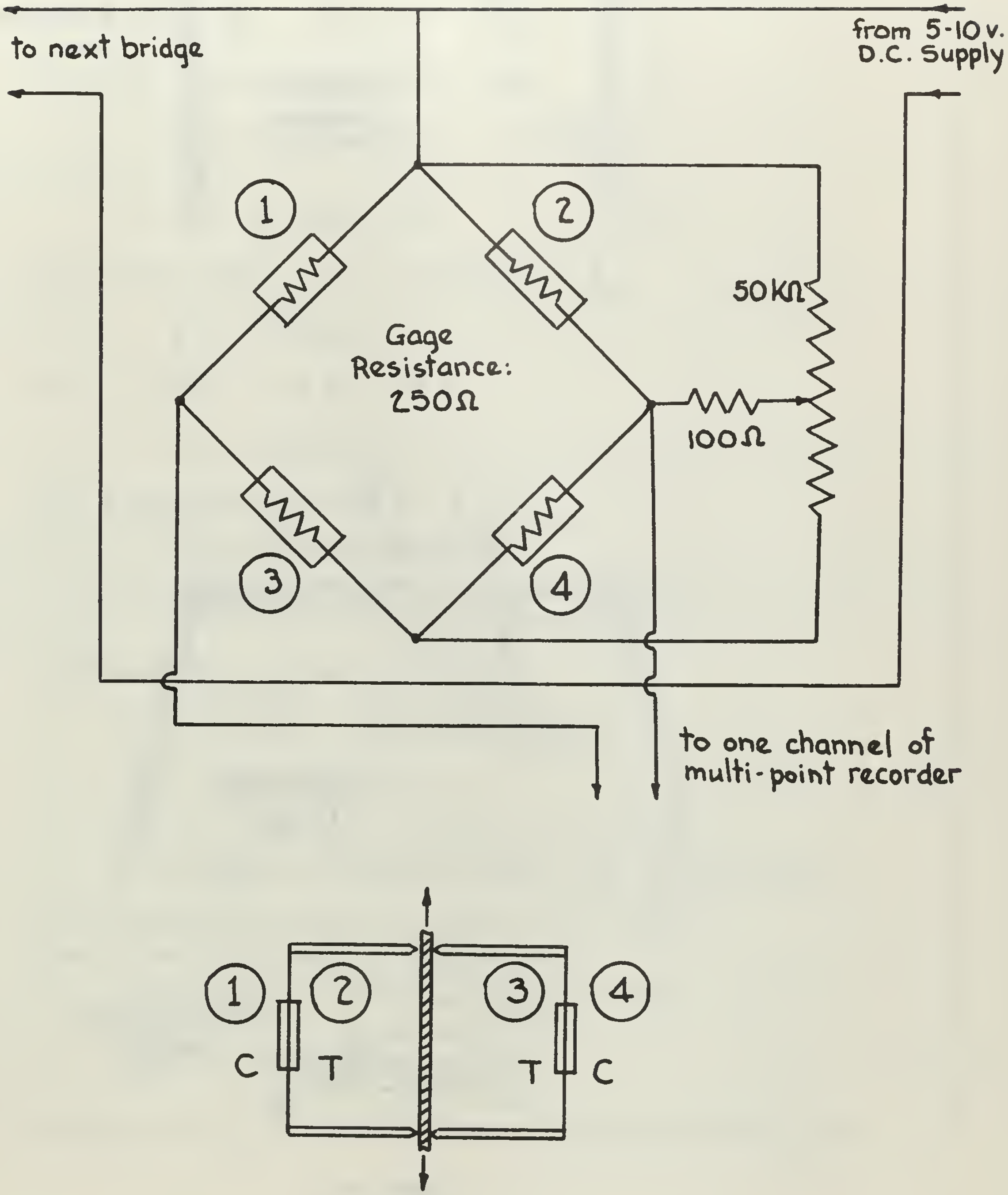


Figure 53

Bridge Circuitry for Extensometer

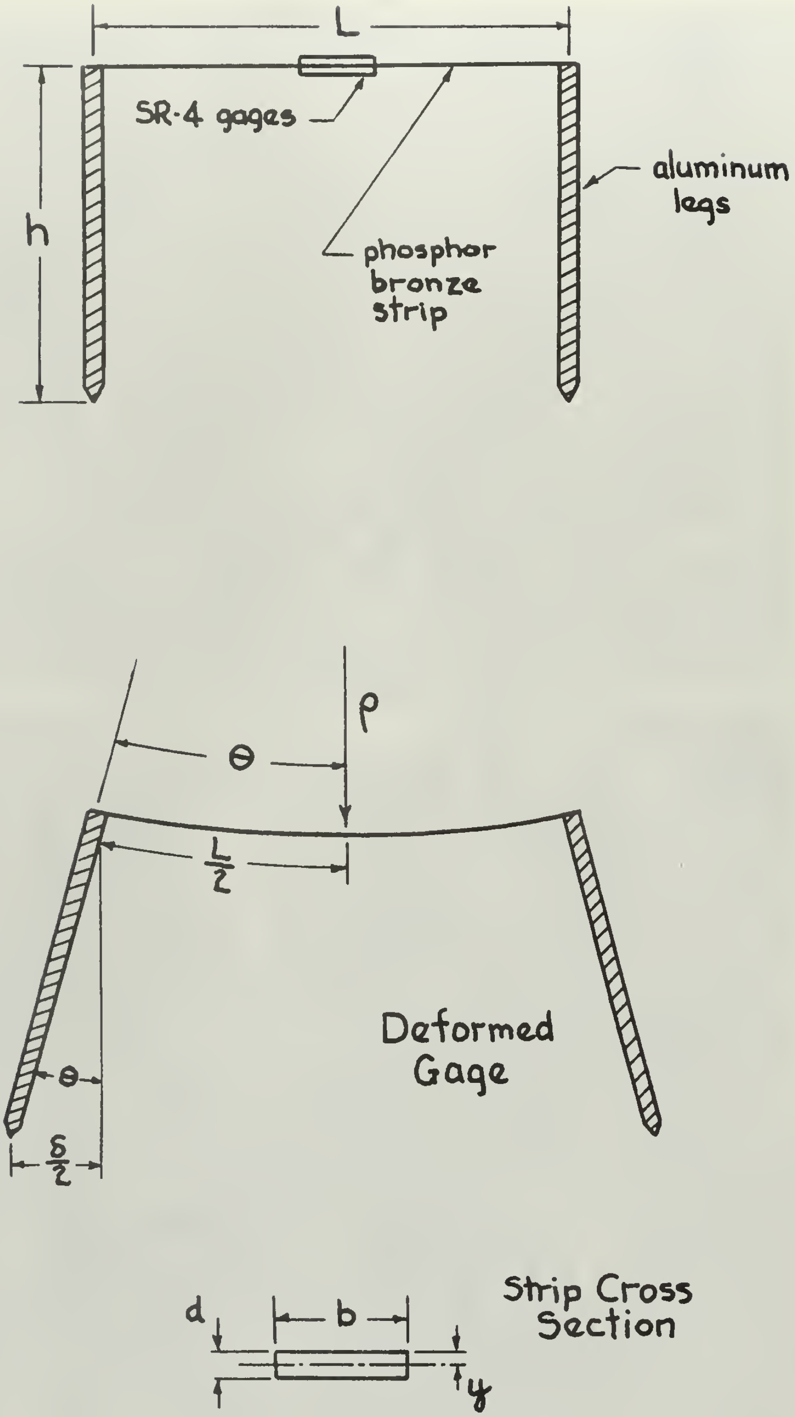


Figure 54

Dimensioned Sketch for Clip Gauge Stress Analysis

CLIP GAUGE STRESS ANALYSIS

- Assume no bending in aluminum legs
- To find ϵ in gauges for a given specimen extension, δ

$$\rho\theta = \frac{L}{2}, \quad \theta = \sin^{-1}\left(\frac{\delta}{2h}\right)$$

from the flexure formula, $\epsilon = y/\rho$

$$\rho = L/2\sin^{-1}\left(\frac{\delta}{2h}\right), \quad y = d/2$$

$$\therefore\ \epsilon = \frac{d}{L}\sin^{-1}\left(\frac{\delta}{2h}\right)$$

for small angles θ, $\sin^{-1}\left(\frac{\delta}{2h}\right) \approx \frac{\delta}{2h}$

$$\epsilon = d\delta/2Lh$$

for $d = .010''$, $h = 2''$, $L = 2''$

$$\epsilon = .00125\,\delta$$

for a specimen extension of $\delta = 1''$

$$\epsilon = 1250\,\mu\text{inches/inch}$$

or about one-half the maximum strain capability of the BLH type a-5-1 gages

maximum stress in the bronze strip:

$$\sigma = E\epsilon, \quad \begin{array}{l} \epsilon = .00125 \\ E = 16 \times 10^6\ \text{psi} \end{array}$$

$$\sigma = 20{,}000\ \text{psi}$$

or only a small fraction of the yield strength of phosphor bronze (σ_y = 112,000 psi)

at an extension of $\delta = 1''$, error in the strain due to the small angle assumption is:

$$\delta/2h = 1/2\cdot 2 = .25, \quad \sin^{-1}\left(\frac{\delta}{2h}\right) = .2474$$

$$\text{Error} = \frac{.25 - .2474}{.25} = 1\%$$

Figure 55

Theoretical Stress Analysis of Clip Gauges

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13. ABSTRACT

Constant load creep data were obtained for four plastics: polyethyline, polypropylene, nylon, and polycarbonate. Viscoelastic responses of these materials proved to be non-linear in both time and stress. An empirical creep equation, based on a time dependent retardation rate constant, was developed to characterize the creep behavior and good correlation was obtained for various stress levels. By assuming the developed relationship to be an equation of state, creep behavior of simple composites of plastics was predicted with good results when viscoelastic responses of the components were similar. The importance of stress-time history is demonstrated by using the creep equation to estimate mechanical behavior in other modes of testing. The concept of a retardation spectrum and its application to the materials studied is considered.

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	ROLE	WT	ROLE	WT	ROLE	WT
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Viscoelastic						
Creep						
Plastic						
Polyethylene						
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